JOURNAL
OF THE
WASHINGTON ACADEMY OF SCIENCES

VOL. 15 JULY 19, 1925 No. 13

PHYSICS.—*Some remarks on two-coupled multiply periodic systems, the statistics of quantum theory, and the theory of dispersion.* G. BREIT, Department of Terrestrial Magnetism, Carnegie Institution of Washington. (Communicated by L. A. Bauer.)

1. The quantum theory postulates the substitution of discrete states which are to replace the continuous manifolds of the classical theory. However, it says very little about the nature of the interactions which take place between quantized systems. In fact, the only cases about which definite postulates have been made are absorption of radiation in quanta by an atom put in black body radiation, which has been treated by Einstein,[1] and the behavior of free electrons put in black radiation. The two cases can be considered as special cases of the postulate of Einstein and Ehrenfest. Similar considerations concerning the nature of interactions between sound and light waves have been made by Schrödinger.[2]

The writer has performed some calculations on a purely classical basis which seem to have a bearing on the nature of actual (quantum theory) interactions. These calculations deal with a somewhat more general case than that considered by Kramers and Heisenberg[3] in their recent paper on the scattering of light and optical dispersion and agree in a general way with their results, though the point of view taken by the writer as well as the conclusions drawn are slightly different.

It has been pointed out by Burgers[4] that the problem of optical dispersion can be reduced to a consideration of coupled systems, and that as a consequence of this the purely formal quantum theory does

[1] A. EINSTEIN, Physik. Zeit., 1917; W. PAULI, *Zeit. f. Physik.*, **18**: 273. 1923; A. EINSTEIN and P. EHRENFEXT, Zeit. f. Physik, **19**: 301. 1923.

[2] E. SCHRÖDINGER, Physik, Zeit., **25**: 89–94. 1924.

[3] KRAMERS and HEISENBERG, Zeit. f. Physik, **31**: 681. 1925.

[4] BURGERS, Dissertation, Leiden.

not account for optical dispersion in a proper fashion, giving rise to absorption bands at frequencies corresponding to the dynamical rather than the quantum values. This fact makes one suspect that the failure of classical perturbation theory in the case of dispersion forms only a special case of a general deficiency, and that by presenting the problem of dispersion in the form of a problem on coupled systems in the manner of Burgers one may hope to generalize the nature of interactions between a light wave and an atom to the case of coupled systems.

2. We consider, therefore, two non-degenerate coupled systems, the angular variables w_i, the momenta J_i, and the energy function H of the first being

$$(w_1, \cdot\cdot, w_n; J_1, \cdot\cdot J_n)\ H_n(J_1, \cdot\cdot J_n)$$

and for the second

$$(w_{n+1}, \cdot\cdot, w_{n+m}, J_{n+1}, \cdot\cdot J_{n+m})\ H_m(J_{n+1}, \cdot\cdot J_{n+m})$$

In the absence of coupling, the motions of the two systems progress independently. The presence of coupling is manifested by the presence of a perturbation term

$$\lambda F\ (J_1, \cdot\cdot J_{n+m}; w_1, \cdot\cdot w_{n+m})$$

in the total energy

$$H = H_n + H_m + \lambda F \qquad (1)$$

The effect of the perturbations may be calculated either by the method of Born and Pauli or by the method used by Van Vleck in studying the interaction of light waves and multiply periodic systems. We prefer in what follows to use the latter's method because it enables one to use the initial conditions of the system very much more readily than the method of Born and Pauli.[5]

The method used by Van Vleck is based on the fact that for the unperturbed system the transformation from the Cartesian coördinates and momenta to the angular variables and their corresponding momenta is a contact transformation, and also upon the theorem that Hamilton's canonical equations are invariant under a contact trans-

[5] In the latter method new variables are introduced at each stage of the approximation. Strictly speaking, it is questionable whether the dispersion problem can be always treated as a multiply periodic system even if the unperturbed system is multiply periodic. The manner in which the new variables w and J depend on initial conditions is not clear unless it is especially determined, and for our present purpose we shall prefer an explicit calculation.

formation. The calculations are performed, therefore, always for quantities w and J which have the same significance in terms of the Cartesian coördinates and momenta throughout. This, of course, is true only provided the transformation function w does not involve the time explicitly.[6] Such is the case for all systems which are conditionally periodic.

It must be mentioned, however, that in another connection the method followed by Born and Pauli possesses certain advantages over that of Van Vleck. Thus, if we are concerned with the condition of the two systems *while* they perturb each other, the method of Born and Pauli enables us to find new angular variables which are suited for the quantization of the whole coupled system. On the other hand, if we are interested in the effect on the two systems of having been exposed to the perturbations arising from their coupling *after* the coupling has been interrupted, then Van Vleck's method is preferable.

If we are interested in a given function of the Cartesian coordinates, both methods may be used and they must both give the same result.

Before discussing our general case we shall illustrate the meaning of the quantities used in the special case of a linear oscillator having one degree of freedom. Here, the energy function H has the expression

$$\mathrm{H} = \frac{\mathrm{p}^2}{2\,\mathrm{m}} + \frac{\mathrm{m}}{2}\,\Omega^2\mathrm{q}^2 \tag{1.1}$$

where q is the abscissa of the oscillating particle and p is the corresponding momentum. The angular variable w is

$$\omega = \frac{1}{2\,\pi}\sin^{-1}\left(\mathrm{m}\,\Omega\,\mathrm{q}/\sqrt{\mathrm{m}\,\mathrm{J}\,\Omega/\pi}\right) \tag{1.2}$$

and the corresponding momentum

$$\mathrm{J} = 2\,\pi\,\mathrm{H}/\Omega \tag{1.3}$$

If we should apply our calculations to this case, they will give us changes in the quantities w, J, where J is obtained from (1.3) with H expressed by means of (1.1) [in terms of q, p] and w is obtained from (1.2) on substituting this value of J. The meaning of w and J in this as well as the general case is thus that of certain explicit functions of w and J, the type of function being, of course, different for different dynamical systems.

[6] E. T. WHITTAKER, *Analytical Dynamics*, p. 309.

It is hardly necessary to state that as soon as the nature of the perturbation function F in (1) is known in terms of the ordinary coördinates q_i and their corresponding momenta p_i, a direct substitution by means of such equations as (1.2) and (1.3) gives the expression of F in terms of the w_i and J_i. Since the coördinates are multiply periodic functions of the w_i with period 1, we may write

$$F = \sum A_\tau \exp 2\pi i w_\tau \tag{2}$$

where

$$A_\tau = A_{\tau_1, \tau_2 \cdots \tau_{n+m}} \tag{1.2}$$

$$w_\tau = \tau_1 w_1 + \tau_2 w_2 + \cdots + \tau_{n+m} w_{n+m} \tag{2.2}$$

and where it is supposed that the τ_2 may take the values zero and all the positive and negative integral values. The A_τ are functions of the J's only, and the w_i may be written as

$$w_i = \omega_i t + \epsilon_i \tag{2.3}$$

where the w_i are functions of the J's only and the ϵ_i are constants. Hamilton's canonical equations are

$$\frac{dJ_k}{dt} = -\lambda \sum 2\pi i A_\tau \tau_k \exp(2\pi i w_\tau) \tag{3}$$

$$\frac{dw_k}{dt} = \lambda \sum \frac{\partial A_\tau}{\partial J_k} \exp(2\pi i w_\tau) + \frac{\partial (H_n + H_m)}{\partial J_k} \tag{4}$$

We expand

$$J_k = J_k^0 + \lambda J_k^{(1)} + \lambda^2 J_k^{(2)} + \cdots \tag{5.1}$$

$$w_k = w_k^{(0)} + \lambda w_k^{(1)} + \lambda^2 w_k^{2(2)} + \cdots \tag{5.2}$$

and substitute in (3), (4). Comparing coefficients of λ, we obtain the system of equations

$$\frac{dJ_k^0}{dt} = 0 \tag{5.31}$$

$$\frac{dJ_k^{(1)}}{dt} = -2\pi i \sum (\tau_k A_\tau \exp 2\pi i w_\tau)_0 \tag{5.32}$$

$$\frac{dJ_k^{(2)}}{dt} = -2\pi i \left[\sum \left(Jl^{(1)} \frac{\partial}{\partial J_l} + w_l^{(1)} \frac{\partial}{\partial w_l} \right) \left(\tau l A_\tau \exp 2\pi i w_\tau \right) \right]_0 \tag{5.33}$$

. . . . . .

and

$$\frac{dw_k^0}{dt} = \omega_{k0} \tag{5.41}$$

$$\frac{dw_k^{(1)}}{dt} = \sum\left(J_l^{(1)} \frac{d\omega_k}{dJ_l}\right)_0 + \sum\left(\frac{\partial A_\tau}{\partial J_k} \exp 2\pi i w_\tau\right)_0 \tag{5.42}$$

. . . . . .

where the suffix o indicates that throughout the unperturbed values of the w and the J must be substituted. Having determined J_k^1 from (5.32) we can substitute it into (5.42), and having then determined w_k^1 we substitute it and J_k^1 into (5.33) which determines J_k^2 etc.

We suppose that the coupling between the two systems has been established at the time

$$t = t_o$$

and find from (5.32) that

$$J_k^{(1)} = \sum \tau_k A_\tau \omega_\tau^{-1} [\exp 2\pi i w_\tau(t_o) - \exp 2\pi i w_\tau(t)] \tag{5.5}$$

Substituting this in (5.42) and integrating we have

$$w_k^{(1)} = \sum\left[\frac{1}{2\pi i}\left(\frac{\partial}{\partial J_k}\left(\frac{A_\tau}{w_\tau}\right)\right)\left(\exp 2\pi i w_\tau(t) - \exp 2\pi i w_\tau(t_o)\right) + \frac{A_\tau}{\omega_\tau}\frac{\partial \omega_\tau}{\partial J_k}(t - t_0) \exp 2\pi i w_\tau(t_0)\right) \tag{5.6}$$

Substituting this and (5.5) into (5.33) and integrating

$$J_k^{(2)} = -2\pi i \sum_{\tau, l, \tau'} \tau_k \left\{\left[\tau_l' \frac{A_{\tau'}}{\omega_{\tau'}}\frac{\partial A_\tau}{\partial J_l} - A_\tau \tau_l \left(\frac{\partial}{\partial J_l}\left(\frac{A_{\tau'}}{\omega_{\tau'}}\right)\right)\right]\left[\frac{\exp 2\pi i w_{\tau'}(t_0)}{2\pi i w_\tau} \times (\exp 2\pi i w_\tau - \exp 2\pi i w_\tau(t_0)) - (2\pi l(\omega_\tau + \omega_{\tau'}))^{-1} (\exp 2\pi l(w_\tau + w_{\tau'})) - \exp 2\pi i(w_\tau(t_0) + w_{\tau'}(t_0)))] + 2\pi i A_\tau A_{\tau'} \omega_{\tau'}^{-1}\left(\tau_l \frac{\partial w_{\tau'}}{\partial J_l}\right)\left(\exp(2\pi i w_{\tau'}(t_0))\right) \times \left\{(2\pi i \omega_\tau)^{-2} (2\pi i \omega_\tau(t - t_0) \exp 2\pi i w_\tau - [\exp 2\pi i w_\tau - \exp 2\pi i w_\tau(t_0)])\right\} \tag{5.7}$$

Thus it is seen that if the coupling has been established at the time $t = t_o$ and interrupted at the time t, then on the classical theory we should expect in general a rather complex result for the resultant change in the state of the system from the one it would have had if it had not been perturbed during that interval.

In the above calculation the phase of the motion has a very strong

effect. This is quite proper from the point of view of classical theory. On the quantum theory, however, we have indications only of statistical interpretations of the phase, and since we want to establish a connection between expectations of classical dynamics and quantum jumps we must eliminate effects of phase in the above results. One way of doing this is to consider *forced vibrations*. Since, however, a treatment of forced vibrations for a multiply periodic system is connected with difficulties such, for example, as the falling of an electron into the nucleus, we prefer to use the Rayleigh-Lorentz method of interruptions which for the case of a classical linear resonator is equivalent to forced vibrations.

We consider a number of systems and we wish to know their average state at the time t. We suppose that perturbations in these systems have begun to take place some time before t and we assume that the selection of the starting time is governed by the laws of chance so that if

$$T = t - t_o \tag{6}$$

then the probability of T being between T and T + dT is

$$e^{-\frac{T}{T_o}} d\left(\frac{T}{T_o}\right) \tag{6.1}$$

We thus find averaging (5.7) that

$$\overline{J_k^{(2)}} = \sum_{\tau, \tau' l} \tau_k \, C(\omega_\tau + \omega_{\tau'}) \left\{ (\tau_l' + \tau_l) \, C(\omega_{\tau'}) A_{\tau'} \frac{\partial A_\tau}{\partial J_l} - \tau_l \frac{\partial}{\partial J_l} (A_\tau A_{\tau'} C(\omega_{\tau'})) \right\} \exp 2\pi i (w_\tau + w_{\tau'}) \tag{7}$$

where

$$C(\omega_\tau) = \frac{2\pi i \, T_0}{1 + 2\pi i \, T_0 \, \omega_\tau} \tag{7.1}$$

This expression for $J_k^{(2)}$ contains periodic as well as constant terms. The constant terms are by far the more important because they correspond to the case

$$w_\tau + w_{\tau'} = 0 \tag{8}$$

$$(\tau_1 = -\tau_1', \tau_2 = -\tau_2', \text{ etc.})$$

and because $C(\omega_\tau + \omega_{\tau'})$ becomes $2\pi i T_o$ which is $> > (\omega_\tau + \omega_{\tau'})^{-1}$ pro-

vided the mean period of interruptions is considerably greater than the important period of the motion. We thus see that terms which give large average effects on J_k may be collected into the approximate sum

$$\overline{J_k^{(2)}} \cong - \sum_{\tau, l} \tau_k \tau_l \frac{\partial}{\partial J_l} (A_\tau A_{-\tau} C(\omega_\tau)) \cdot 2\pi i T_0 \qquad (9)$$

Here again, only those terms are great for which

$$\omega_\tau \cong 0 \qquad (10)$$

This may be interpreted as a resonance of one part of the dynamical system to another (approach of degeneration). In the case of the influence of a light wave on an atom, this means the approximate equality of the frequency ν of the incident light wave to an overtone w_τ of the atom.

3. According to the view of Burgers referred to above, the problem of optical dispersion is mathematically equivalent with that of a coupled system. We should like to emphasize the importance of this view also in another connection. It is well known that in classical electrodynamics there are two ways of discussing interactions between electrical systems. One simply makes use of the supposedly known laws connecting the electric and magnetic fields with the distribution of charges and currents. The other invokes the concepts of electromagnetic energy and momentum. The two ways are equivalent and not at all exclusive. Are there analogies to both of these procedures on the quantum theory? So far it has been customary to deal only with the second (field energy, field momentum) point of view. If, however, the direct point of view be also manageable, we must be able to treat the following problem:

A sending atom S is coupled to a receiving atom R by means of the dispersing atom D. Required: The effect of D on the absorption of R.

The state of the medium between S, D, and R matters only inasmuch as it effects R, and this is always true for actual observation because all experiments deal ultimately with material particles and because our observation of a field always depends on experiments with matter. On the classical theory nothing *need* be said about the electromagnetic energy or momentum in discussing this problem. Similarly we may suspect that if a proper formulation of quantum theory be found, the same would be true for it as well.

Since quantum theory is concerned with sudden transitions in the J's, it seems natural to consider our problem from the point of view of probabilities of transitions in the quantum states of R. Formula (9) gives the main contributions to steady changes in J_k which we would expect on the classical theory. In order to apply this formula to our case we consider our system symbolically as

$$((S) + (R)) + (D)$$

meaning by this that (D) is first thought of as separated from $(S) + (R)$ and then its coupling is taken into account by means of (9).

Now in calculating the effect of D on R it is obviously useful to introduce the auxiliary quantity called the electric moment of D, but otherwise it has no significance. It may be shown using Van Vleck's calculations in a manner analogous to that of § 2 (or else considering the problem as a special case of § 2) that if the wave due to S is of the form $E_x\cos 2\pi(\nu t - \epsilon_0)$ and D is a triply periodic atom having a charge $-e$, then the averaged changes of the first order in E_x caused by the wave in D are

$$\overline{\Delta_1 J_k} = -\pi i e\, E_x \sum \frac{T_0}{1 + 2\pi i(\omega_\tau \mp \nu)T_0} \times \tau_k A_\tau \exp 2\pi i(\omega_\tau t \mp \nu t + \epsilon_\tau \mp \epsilon_0) \tag{10}$$

$$\overline{\Delta_1 H} = -\pi i e E_x \sum \frac{T_0}{1 + 2\pi i(\omega_\tau \mp \nu)T_0} \times \omega_\tau A_\tau \exp 2\pi i(\omega_\tau \mp \nu t + \epsilon_\tau \mp \epsilon_0) \tag{11}$$

and letting us use our previous notation for $C(\omega_\tau)$, the second order contributions are

$$\overline{\Delta_2 J_k} = -\frac{e^2 E_x^2}{4} \sum_{\tau,\tau',l} \tau_k\, C(\omega_\tau \mp \nu + \omega_{\tau'} \mp \nu) \left\{ \tau_l \frac{\partial (A_\tau\, A_{\tau'}\, C(\omega_{\tau'} \mp \nu)}{\partial J_l} - (\tau_l + \tau_l')\, C(\omega_{\tau'} \mp \nu)\, A_{\tau'} \frac{\partial A_\tau}{\partial J_l} \right\} \exp 2\pi i \left((\omega_\tau + \omega_{\tau'} \mp \nu \mp \nu)t + \epsilon_\tau + \epsilon_{\tau'} \mp \epsilon_0 \mp \epsilon_0 \right) \tag{12}$$

$$\overline{\Delta_2 H} = -\frac{e^2 E_x^2}{4} \sum C(\omega_\tau \mp \nu + \omega_{\tau'} \mp \nu) \left\{ \omega_\tau\, \tau_l\, A_\tau \frac{\partial (A_{\tau'} C(\omega_{\tau'} \mp \nu))}{\partial J_l} - \omega_\tau\, C(\omega_{\tau'} \mp \nu)\, \tau_l'\, A_{\tau'} \frac{\partial A_\tau}{\partial J_l} - \frac{A_\tau\, A_{\tau'}}{2} [C(\omega_\tau \mp \nu) + C(\omega_{\tau'} \mp \nu)] \times \right.$$

$$\tau_l \frac{\partial \omega_{\tau'}}{\partial J_l} \Big\} \exp 2\pi i[(\omega_\tau + \omega_{\tau'} \mp \nu \mp \nu)\, t + \epsilon_\tau + \epsilon_{\tau'} \mp \epsilon_0 \mp \epsilon_0] \qquad (13)$$

At the same time the average change in the x coordinate of the electron is

$$\overline{\Delta x} = \frac{eE_x}{2} \sum \Big[\tau_l \frac{\partial}{\partial J_l} (C(\omega_{\tau'} \mp \nu)\, A_\tau A_{\tau'}) - (\tau_l + \tau_l')\, C(\omega_{\tau'} \mp \nu) \times$$

$$A_{\tau'} \frac{\partial A_\tau}{\partial J_l} \Big] \exp 2\pi i\, [(\omega_\tau + \omega_{\tau'} \mp \nu)\, t + \epsilon_\tau + \epsilon_{\tau'} \mp \epsilon_0] \qquad (14)$$

The quantity $\overline{\Delta x}$ when multiplied by $-e$ give the average electric moment.

The structure of the terms in (12) and (14) is very similar. The terms for which $\omega_\tau + \omega_{\tau'} = 0$ in (14) give contributions to the polarization which are synchronous with the incident wave. As remarked by Kramers and Heisenberg and as has been first published by Van Vleck, other terms than those of frequency ν are present in the polarization. Kramers and Heisenberg attach a real significance to such terms for which $\omega_\tau' - \nu \doteq 0$. Numerically such terms are great. However, we must note that the corresponding terms in (12) give periodic contributions to $\overline{\Delta_2 J_k}$. It may be that this must be considered a reason for not attaching a physical significance to such terms because they do not give rise to anything reminding one of a quantum jump.

We may be permitted to say now a few words about the theory of optical dispersion advanced by Kramers and the speculations of Born[7]) on the same subject.

We should like to point out that the substitutions of $\Gamma(n + \tau, n)$ for $[C_\tau]^2$ and of $\tau\,(n + \tau, n)$ for (τ_τ) are arbitrary (especially the latter). These substitutions involve an element not included in Bohr's original correspondence principle, the spirit of which is given by Born's formula (29). The original idea seemed to be to obtain quantum quantities from classical quantities by taking proper averages. Kramers's formula regards a quantum quantity (the electric polarization) as an average of a combination of quantum quantities, the most important of these being the actual emission or absorption frequency. Another difference between the procedure of Kramers and older applications of the correspondence principle lies in the fact that the older applications dealt always with an actual jump of the atom. This is so in the calculations of frequency, intensities of lines and selection rules.

[7] M. Born, Zeit. f. Physik, **26**, p. 379 and especially pp. 389, 390, 394.

In the application of Kramers the negative virtual oscillators are given a prominent role because it is desired to use the results for atoms which do not perform jumps. Is not this analogous to interpreting the fundamental formula

$$\nu = \frac{W_2}{h} - \frac{W_1}{h}$$

as meaning that in the lowest state the atom emits a negative frequency

$$-\frac{W_1}{h}?$$

It seems to the writer to be worth while to consider the meaning of such formulas as (9) and (12) so far as possible extensions to quantum theory are concerned. This has been done by Born (l. c.) in his formula (33). The obvious similarity of (9) and (12) and the connection of (12) with Einstein's B_m^n terms make one desire to have a similar interpretation of (9). Such an interpretation must satisfy the laws of statistical mechanics if we generalize Einstein's A_m^n correctly.

If Born's extension (33) is correct, we have the same state of things in an elastic body coupled to an atom as we have in Einstein's original problem. If such is the case, we must surmise the general applicability of Einstein's scheme to such cases.

The relation of this to Born's point of view may be seen from the fact that Einstein's B_m^n may then be thought of as desirable on the Correspondence Principle from (12) and in the generalized case from (7) and (9). This involves an integration over a spectrum of ν in (12). We know physically that the singularities of (12) are such values of ν which are equal to the absorption (quantum) frequencies and not to the classical frequencies. Therefore we must similarly suppose in (7) and (9) that if our H_n and H_m systems have two nearly equal quantum jumps a large probability of interaction results and that this probability is given by a dispersion relation the true meaning of $\frac{1}{\nu-\nu_0}$ in a dispersion of formula being $\frac{h}{h\nu - h\nu_0} = \frac{h}{\Delta W - \Delta W_0}$.[8]

[8] Dr. A. E. Ruark, in a conversation with the writer mentioned that such a possibility of interpreting ν has also occurred to him as being a possible one *a priori*.

PHYSICS.—*Note on the use of a light filter in interferometry.* JOHN BRIGHT FERGUSON, Department of Chemistry, University of Toronto.

The Rayleigh-Zeiss type of interferometer[1] is the type most frequently used in chemical work. It may be used either as a zero instrument or as a measuring instrument, and for most purposes white light is employed since the chromatic bands formed by the latter enable one to identify the reference band.

When the present writer used a tungsten lamp as a source of light, it was noted that the eyes soon tired and that the identification of the bands became a difficult task. This source of trouble was largely eliminated by the use of a light filter which absorbed a large part of the yellow and blue light. The potassium dichromate, neodymium nitrate liquid filter worked very well. The effect of this filter is to decrease the illumination of the back ground and to make the green and red portions of the chromatic bands purer in color.

Visual acuity is known to be increased by the use of monochromatic light and to decrease as the brightness of the background increases within certain limit,[2] so that the beneficial effect of the filter is quite in accord with our present-day knowledge of the factors affecting visibility.

CHEMISTRY.—*A Magnetic form of ferrous oxide.* JOHN BRIGHT FERGUSON, Department of Chemistry, University of Toronto.

Metallic iron undergoes a magnetic change when it is heated and above 790°C., it is no longer magnetic.[1] Similarly ferric oxide possesses a magnetic inversion at 678°[2] and ferro-ferric oxide also an inversion of like character at 530°[3]. If the magnetic properties are atomic in character,[4] ferrous oxide might therefore be expected to show a magnetic inversion in the same temperature region.

Pure ferrous oxide was prepared for the first time in 1921 by Wöhler and Balz.[5] They report that their product was non-magnetic in character. Our own products[6] were also non-magnetic in character. They were prepared above 749° and contained ferro-ferric oxide. In our study of the transformation of these phases into free iron and ferro-ferric oxide and the subsequent recombination of the reaction products, we noted the following abnormal behavior. During the

[1] For a description of this instrument see L. H. ADAMS, This JOURNAL, **5**: 265. 1915, or Journ. Amer. Chem. Soc. **37**: 1181. 1915.

[2] LUCKIESH, *Color and its applications*, New York, 1921 and Elliott, Amer. Journ. Psych. **33**: 97. 1922.

[1] HONDA, Science Reports, Tohoku Imp. Univ. **11**: 119. 1922.

[2] SOSMAN and HOSTETTER, Journ. Amer. Chem Soc. **38**: 807. 1916.

[3] SOSMAN, This JOURNAL, **7**: 55. 1917.

[4] Honda, loc. cit.

[5] WÖHLER and BALZ, Zeitschr. Electrochem. **27**: 406. 1921.

[6] FERGUSON, This JOURNAL, **13**: 275. 1923.

recombination of the free iron, the decrease in the magnetic permeability did not correspond to the decrease in the free iron content of the sample. Charges containing small amounts of free iron were quite magnetic after heating for five hours at 610°, but after a similar heat treatment at 630° the magnetic property largely vanished. In one case the behavior was of this character though the charge contained no free iron at either temperature. These results would seem to indicate that there is a form of ferrous oxide which is magnetic in character. Whether this form has a stability range from about 630° to the transition temperature at approximately 570°, or whether it occurs as an unstable phase, is a question which cannot now be answered. The probable occurrence of a magnetic inversion in ferrous oxide which is here indicated, is of itself an interesting point when considered with the magnetic behavior of iron and the other oxides of iron.

PETROLOGY.—*The mineralogical phase rule.* N. L. BOWEN, Geophysical Laboratory.

In 1911 V. M. Goldschmidt, in connection with his study of contact metamorphism in the Kristiania region, enunciated a mineralogical phase rule which was stated by him as follows: "The maximum number of solid minerals that can co-exist in stable equilibrium is equal to the number of individual components that are contained in the minerals if the singular temperatures of transition points are excluded."[1]

Goldschmidt developed it by consideration of solid phases in contact with their saturated solution, a method which has the advantage that it is probably a real picture of what goes on in contact metamorphic processes. It may be developed in a somewhat more general form which includes transformations for which the aid of a solution need not be postulated. Thus the phase rule states that the number of phases plus the number of degrees of freedom exceeds the number of components by 2

$$p + f = c + 2$$

Having regard for conditions under which only solid phases are present, and taking for illustration a 3-component system, we could have at most 5 solid phases. This condition could occur only when the temperature and pressure had each a definite value, and since the probability is very small that the conditions of metamorphism of any given

[1] *Die kontaktmetamorphose im Kristianiagebiet.* Vid.-Selsk. Skrifter I. Math. Naturw. Kl. No. 1, p. 125. 1911.

rock were exactly those corresponding to these values we may not expect to find a 3-component rock containing 5 solid phases in equilibrium. If there were only 4 solid phases present the system would be univariant, that is, there would be a range of temperatures and pressures at which this would represent an equilibrium condition, but for any definite temperature there would be a definite corresponding pressure at which the 4 solid phases could exist. Again, the random conditions of metamorphism cannot be expected to show for any temperature the requisite precise corresponding pressure. We may not, therefore, expect to find 4 solid phases in a three-component rock at equilibrium. On the other hand, if there are only 3 solid phases present, the system is divariant, that is, for any definite pressure there is a range of temperatures and for any definite temperature, a range of pressures at which this condition can exist. The random conditions attendant upon any particular example of metamorphism cannot but meet these conditions. Therefore the maximum number of solid phases that may be expected in a 3-component rock is 3. The 3-component rock has been chosen merely by way of illustration, and it is easy to see that whatever the number of components chosen it would be found that the maximum number of solid phases to be expected at equilibrium under any random conditions of metamorphism is equal to the number of components. A greater number of phases, assuming that they are in actual contact, indicates a failure to obtain equilibrium. So far the assumptions involved are thoroughly justifiable and though their application to rocks has been criticized adversely, for example, by Johnston and Niggli, actual experience with metamorphic rocks has afforded a very considerable degree of confirmation of the results to be expected, and the mineralogical phase rule has been a very useful guide in the study and classification of metamorphic rocks in the hands of a number of workers, among whom may be mentioned Goldschmidt himself, Eskola, and Tilley.

Since the number of components actually present in a rock is usually rather large and since systems of many components are difficult to treat, especially graphically, certain simplifying assumptions as to the number of components have been made by some writers. These assumptions are based in part on the fact that under certain conditions two or more of the true components may enter into solid solution and give rise therefore to only one phase. Thus Mg_2SiO_4 and Fe_2SiO_4 form a complete series of mix crystals in the olivines, and though these two molecules may be present in a system in amounts varying inde-

pendently of each other they will form but a single phase and therefore can be grouped together as a single component for the purposes of the mineralogical phase rule. At first sight this assumption seems justifiable, but, as we shall see, it must be used with caution.

Another assumption, which, if not definitely stated, is at least implied in the writings of some investigators, is that even if the two molecules do not form a complete series but only a limited series of solid solutions, so long as the concentrations involved do not lie beyond these limits the two molecules may be grouped as a single component. This again seems justifiable since they might be expected to contribute but one phase, but it cannot be justified in the ultimate analysis.

We may illustrate this fact for the latter case by reference to a determined equilibrium diagram for silicates, a simple one of two components, namely, that of nephelite and anorthite.[2] Nephelite shows an inversion at 1248° where it changes to carnegieite. Both nephelite and carnegieite are capable of taking anorthite into solid solution, nephelite up to 35 per cent and carnegieite up to 5 per cent. According to the assumption frequently made in the study of metamorphic rocks any mixture in which the amount of anorthite did not exceed 5 per cent could be regarded as a one-component mixture since it would give rise to but a single phase except at the transition "point." What is actually found is that there is no transition point but a transition interval which, in the mixture containing 4 per cent anorthite, for example, extends from 1270°–1350°. It could not be guaranteed that any random metamorphic process did not take place under conditions falling within this range of 80° where the mixture consists of two solid phases. Therefore the assumption would not be justified that, for all compositions which do not exceed the limit of solution of anorthite in nephelite or carnegieite (or, stated in another form, the limit of replaceability of soda by lime) only one phase could be expected in the hypothetical metamorphic rock. The grouping of nephelite and anorthite (or of CaO and Na_2O) as a single component would therefore be justified only as a convenience. While in the great majority of examples no discrepancy would be encountered, nevertheless when a phase too many was found it would not necessarily be the result of failure of equilibrium but might more properly be referred to the inadequacy of the assumption involved in grouping the true components.

No metamorphic rock is known approaching the mixture referred to

[2] N. L. BOWEN. Amer. Journ. Sci. **33**: 551. 1912.

above, which has, indeed, been chosen merely to illustrate the principle involved. There are, however, many cases where the principle applies. Thus, pyroxene and amphibole no doubt show a relation not unlike that existing between the two phases discussed above, and in some rocks containing these minerals the number of solid phases found is too great for the number of (assumed) components. At times this fact has been considered to indicate lack of equilibrium, one of the phases being regarded as an "unstable relic,"[3] but in many of these cases there may be no such failure of equilibrium. The difficulty may, in reality, arise from the choice of too small a number of components. The grouping of, say, ferrous iron and magnesia as a single component may lead to no difficulties in the great majority of rocks, but in those few that are formed under conditions that lie within what may be termed inversion intervals, a number of phases too great for agreement with such grouping may be found even with perfect equilibrium.

Another limitation should be noted upon the simplification made by grouping the true components in those cases where solid solution occurs. In the three component system MgO-Al_2O_3-SiO_2 there is, no doubt, some range of temperature in which the three solid phases sillimanite, spinel, and corundum can exist together. The temperatures concerned were formerly believed to be those immediately below liquefaction, but this cannot now be maintained since it has been shown in both synthetic mixtures and natural rocks that mullite, $3Al_2O_3 \cdot 2SiO_2$, and not sillimanite is the compound of alumina and silica formed at such temperatures.[4] Nevertheless, natural rocks do show the association noted above and it is unquestionably a stable one under the conditions concerned. There is also, no doubt, another range of temperature in which cordierite, corundum, and sillimanite can coexist. The whole matter has been discussed very fully by Tilley,[5] to whose paper the reader is referred for details. These two different three-phase assemblages indicate, as Tilley has shown, the possibility of the reaction:

$$2(MgO \cdot Al_2O_3) + 5(Al_2O_3 \cdot SiO_2) \rightleftharpoons 2MgO \cdot 2Al_2O_3 \cdot 5SiO_2 + 5Al_2O_3$$

spinel sillimanite cordierite corundum

[3] A very useful term which we owe to ESKOLA. Norsk Geologisk Tidskrift, **6**: 149. 1920.

[4] BOWEN and GREIG, Journ. Amer. Ceram. Soc. **7**: 238–254. 1924, and BOWEN, GREIG and ZIES, This JOURNAL **14**: 183–191. 1924.

[5] C. E. TILLEY, Geol. Mag., **60**: 101–107. 1923.

All four solid phases can occur together at any given pressure only at a definite temperature, so we may not expect to find all four present at equilibrium in any rock containing only these oxides. We will find one of the four possible three-phase assemblages according to whether the temperature at which the condition was established was above or below the reaction temperature. Now, if ferrous iron were also present, and whether it may replace the magnesia of the two magnesian compounds to an unlimited extent or only partially, the relations will be changed.

We will then be concerned with the reaction:

$$2[(Mg,Fe)O\cdot Al_2O_3] + 5(Al_2O_3\cdot SiO_2) \rightleftharpoons$$

spinel sillimanite

$$2(Mg,Fe)O\cdot 2Al_2O_3\cdot 5SiO_2 + 5Al_2O_3$$

cordierite corundum

This reaction will necessarily take place through a range of temperatures at any given pressure or through a range of pressures at any given temperature. Given a certain pressure there will be for any temperature within the range a definite composition for each of the two phases of variable composition, but all four phases with appropriate adjustments of composition may exist together throughout a range of conditions. Again we may not safely assume that the conditions attendant upon any specific example of the metamorphic processes did not fall within this range and we might therefore expect at times to find the four phases together even in a rock which had attained complete equilibrium under the conditions of its formation. Thus, Tilley notes in the Comrie area of Scotland the frequent occurrence of the four-phase assemblage spinel, sillimanite, cordierite, and corundum and appears to consider that it represents failure of equilibrium.[6] It is to be noted, however, that the four phases could occur together throughout a range of conditions rather than merely at a point and there is no reason why these conditions might not be met in nature.

In summary, it may be stated that in all attempts to decide, on the basis of the number of phases, whether equilibrium has been attained in a metamorphic rock it is essential to bear in mind the fundamental assumptions as to the number of components.

[6] C. E. TILLEY, Quar. Journ. Geol. Soc., **80**: 68, 69. 1924.

BOTANY.—*New plants from Central America—III.*[1] PAUL C. STANDLEY, U. S. National Museum.[2]

The new species described in the present paper were all collected by the writer in 1924 in Costa Rica and Panama. Most of them belong to the Rubiaceae, and chiefly to the genus *Psychotria*, the largest and most difficult of the American genera of this family.

Xylopia xylopioides (Dunal) Standl.

Unona xylopiodes Dunal, Monogr. Anon. 117. *pl. 21.* 1817.
Xylopia grandiflora St. Hil. Fl. Bras. Merid. **1**: 40. 1825.
Xylopia longifolia A. DC. Mém. Soc. Genève **5**: 210. 1832.

Martius (Fl. Bras. **13**[1]: 44. 1841) states that the three names here listed relate to the same plant. If this is true, it is necessary to employ for it the oldest specific name.

Colubrina heteroneura (Griseb.) Standl.

Zizyphus heteroneurus Griseb. Bonplandia **1858**: 3. 1858.
Rhamnus biglandulosa Sessé & Moc. Pl. Nov. Hisp. 38. 1887.
Cormonema nelsoni Rose, Contr. U. S. Nat. Herb. **3**: 315. 1895.
Cormonema biglandulosa Standl. Contr. U. S. Nat. Herb. **23**: 718. 1923.
Cormonema multiflora T. S. Brandeg. Univ. Calif. Publ. Bot. **10**: 411. 1924.
Rhamnus gonzalezii Riley, Kew Bull. **1923**: 173. 1923.

This species was first described from Panama, where it is abundant, but it ranges northward along the Pacific coast to Sinaloa. It is referable to the genus *Cormonema*, published by Reissek, but that genus has no character by which it may be distinguished from *Colubrina* except the presence of spines upon the branches. This is scarcely to be considered a valid generic character.

Doliocarpus multiflorus Standl., sp. nov.

Large woody vine; leaves short-petiolate, the blades broadly obovate to oblong-obovate, 13–21 cm. long, 5–10 cm. wide, obtuse to attenuate at base, acute at apex or rounded and short-acuminate, remotely and coarsely sinuate-serrate toward the apex, coriaceous, bright green, smooth, sparsely puberulent beneath along the nerves or glabrous; flowers long-pedicellate or rarely subsessile, in densely clustered, few-flowered racemes on old wood, the pedicels puberulent; sepals unequal, 3–4 mm. long, glabrous, ciliolate; ovary densely puberulent.

Type in the U. S. National Herbarium, no. 1,219,187, collected on brushy slope near Catival, Province of Colón, Panama, near sea level, January 9, 1924, by Paul C. Standley (no. 30285). *Fendler* 50 from Chagres represents the same species.

This is the only species of *Doliocarpus* with racemose inflorescence known from Central America. In Central America this genus of the Dilleniaceae has been collected only in Panama, where five species are known, but one species grows in Mexico.

[1] See this JOURNAL **15**, 101–107. 1925.

[2] Published by permission of the Secretary of the Smithsonian Institution.

Doliocarpus punctatus Standl., sp. nov.

Large woody vine; petioles 8–12 mm. long; leaf blades oblong to oblong-elliptic, 6–15 cm. long, 2–6.5 cm. wide, usually long-acuminate or sometimes short-acuminate, rounded to acute at base, coarsely sinuate-serrate or subentire, coriaceous, glabrous above, beneath with a few short appressed hairs along the nerves, densely white-punctate; pedicels slender, much longer than the flowers, densely clustered in the axils of the leaves, puberulent, often 2 cm. long; sepals very unequal, brownish, minutely and rather sparsely sericeous, the inner about 8 mm. long; petals white; anthers oblong; fruit globose, red, 1 cm. in diameter, hirtellous-scabrous.

Type in the U. S. National Herbarium, no. 2293, collected near Chagres, Panama, February, 1850, by A. Fendler (no. 305). The following collections also belong here:

PANAMA: Near Catival, Province of Colón, *Standley* 30323. France Field, Canal Zone, *Stevens* 1337. Agua Clara, Canal Zone, *Pittier* 3988. Río Fató, Province of Colón, *Pittier* 3949, 4187. Gatún, *Hayes* 22. Near Puerto Obaldía, San Blas Coast, *Pittier* 4339.

Related to **Doliocarpus dentatus** (Aubl.) Standl. (*Tigarea dentata* Aubl. Pl. Guian. 920. *pl. 351.* 1775), but in that species, which occurs in Panama, the leaves are not punctate, and the ovary is glabrous.

Hybanthus anomalus (H. B. K.) Standl.

Ionidium anomalum H. B. K. Nov. Gen. & Sp. **5**: 381. *pl. 500.* 1821.

Eugenia zetekiana Standl., sp. nov.

Shrub 2.5–4.5 m. high, with few branches, the branchlets densely ferruginous-tomentose; petioles very stout, 6–12 mm. long, densely ferruginous-tomentose; leaf blades lanceolate or narrowly oblong-lanceolate, about 30 cm. long, 6.5–9.5 cm. wide, attenuate at apex, rounded at base, coriaceous, glabrous above, beneath tomentose along the costa or glabrate, the venation prominent on both surfaces, the costa stout, the lateral nerves about 20 on each side, divaricate, slightly curved, anastomosing to form a distinct nerve about 2 mm. from the margin; flowers in terminal, simple or branched racemes, these 7–8 cm. long, few-flowered, densely reddish-velutinous, the flowers subsessile; bractlets minute, subulate; fruit subglobose, 2-celled, 1.5–1.8 cm. in diameter, densely velvety-tomentose; calyx limb short-produced beyond the fruit, persistent, 5-lobate, the lobes semiorbicular, broadly rounded at apex; seeds one in each cell, hemispheric, 12 mm. long, brown and shining.

Type in the U. S. National Herbarium, no. 1,217,607, collected in wet forest on hills north of Frijoles, Canal Zone, Panama, December 19, 1923, by Paul C. Standley (no. 27503). Collected also on hills west of the Canal, near Gatún, *Standley* 27192.

The true position of the plant is somewhat doubtful. It belongs to the genus *Eugenia* as limited by Bentham and Hooker. It has a strong resemblance in aspect to the Brazilian *Schizocalyx pohlianus* Berg, but I doubt that it could be referred to the latter genus, should that group be segregated.

Eugenia zetekiana is named in honor of Mr. James Zetek, who has done more than any other person to popularize the advantages of the Canal Zone as a field for natural history studies. He has spared no personal effort in making the resources of the Zone available for the use of students, and no investigator who has worked in that region can fail to appreciate the results of his labors.

Watsonamra alfaroana Standl., sp. nov.

Simple shrub 1–1.5 m. high, the stems obtusely quadrangular or subterete, about 1.5 cm. in diameter; stipules broadly oblong, 5–5.5 cm. long, very obtuse, brown, finely striate, glabrous within, thinly sericeous outside; petioles 8–20 cm. long, naked below, narrowly winged above the middle, thinly sericeous or glabrate; leaf blades broadly ovate, 40–50 cm. long, about 40 cm. wide, when young sparsely pilose above with minute appressed hairs, beneath densely short-sericeous when young, much of the pubescence persistent in age, the blades pinnatifid two-thirds the distance to the midrib, the segments again pinnatifid, the lobes broadly ovate-oblong, acute, the terminal segment less deeply lobed or merely angulate; cymes dense, few-flowered, sessile or nearly so; fruit globose, 12–15 mm. in diameter, densely tuberculate, capped by the withered calyx; calyx in fruit 2 cm. long, the lobes oblong, obtuse, united below; seeds numerous, brown, acutely angulate.

Type in the U. S. National Herbarium, no. 1,152,712, collected in moist forest along the Río Grande de Tárcoles, near Capulín, Costa Rica, altitude about 80 meters, April 2, 1924, by Paul C. Standley (no. 40194).

Related to *W. gymnopoda* Standl., of the Atlantic coast of Panama, but in that species the leaves are merely once pinnatifid, with narrow, nearly glabrous segments, and the stipules are acuminate or attenuate.

The species is named for Don Anastasio Alfaro, Director of the National Museum of Costa Rica, one of the most distinguished of Central American scientists. To him the writer is indebted for many favors received during the course of botanical work in Costa Rica, among others a delightful visit to the impressive Volcano of Poás.

Psychotria tonduzii Standl., sp. nov.

Simple shrub, about 1 m. high, the stems green, glabrous; stipules green, broadly triangular, about 4 mm. long, at least the base persistent; petioles 2.5–7 cm. long; leaf blades elliptic to obovate-oblong, mostly 19–35 cm. long and 7–14 cm. wide, obtuse or rounded at apex and abruptly contracted into a short broad acute acumen, acute to long-decurrent at base, rather fleshy, glabrous, deep green above, paler beneath; flowers cymose-paniculate, the panicles axillary, usually sessile and branched from the base, sometimes short-pedunculate, with few stout puberulent branches, the flowers sessile in dense heads; bracts and bractlets broad, green, surpassing the calyx; hypanthium puberulent, 1.5 mm. long; calyx 1.5 mm. long, green, puberulent, the 5 lobes broadly ovate, acutish; corolla creamy white, 3 mm. long, minutely puberulent outside, the lobes about equaling the limb.

Type in the U. S. National Herbarium, no. 1,153,160, collected in wet forest at La Estrella, Province of Cartago Costa Rica, March 27, 1924, by Paul C. Standley (no. 39461). The following specimens also belong here: COSTA RICA: Orosi, Province of Cartago, *Standley* 39797, 39850. El Muñeco, south of Navarro, Province of Cartago, alt. about 1,400 meters, *Standley* 33585, 33658, 33439. Boca de Zhorquín, Talamanca, *Tonduz* 8621. La Hondura, Province of San José, alt. about 1,400 meters, *Standley* 36490, 37789.

Related to *P. anomothyrsus* Schum. & Donn. Smith and *P. aggregata* Standl., both of which have long-pedunculate inflorescence.

Psychotria torresiana Standl., sp. nov.

Shrub, about 3 m. high, much branched, the branches slender, green, glabrous; stipules 4 mm. long, green, persistent, closely sheathing, emarginate at apex with a deep broad sinus; petioles slender, 1–2 cm. long; leaf blades lance-linear, 9–16 cm. long, 1.5–2.5 cm. wide, long-acuminate and usually subfalcate at apex, acute or attenuate at base, thin glabrous, deep green above, pale beneath; inflorescence terminal, open-paniculate, many-flowered, slender-pedunculate, about 11 cm. long and 9 cm. wide, the primary branches divaricate or refracted, glabrous, the flowers sessile or short-pedicellate in few-flowered slender-pedunculate cymes, the bracts linear-lanceolate, green, the bractlets minute; calyx 1 mm. long, shallowly 5-dentate; corolla not seen; fruit subglobose, blue, 5 mm. long, the 2 stones coarsely costate dorsally.

Type in the U. S. National Herbarium, no. 1,153,178, collected in wet forest near Orosi, Province of Cartago, Costa Rica, March 30, 1924, by Paul C. Standley (no. 39769). Nos. 39725 and 39883 from Orosi also represent the species.

Easily recognized among the Costa Rican species of the genus by the very narrow leaves. This species is named for Prof. Rubén Torres Rojas to whose courtesy I owe several pleasant excursions.

Psychotria jimenezii Standl., sp. nov.

Shrub or small tree 2.5–4.5 m. high, the branchlets terete, ferruginous-tomentose; stipules triangular, acute, 1 cm. long, brown, early deciduous from the base, tomentulose; petioles 3 mm. long or less; leaf blades oblong-oblanceolate, 9–14.5 cm. long, 2.5–4 cm. wide, long-acuminate, attenuate to the base, deep green above, glabrous, beneath paler, puberulent along the nerves; inflorescence terminal, cymose-paniculate, long-pedunculate or often branched from the base, open, many-flowered, about as broad as long, the branches fulvous-tomentulose, divaricate or ascending at a wide angle; flower sessile in few-flowered headlike cymes; bracts and bractlets small, deciduous; calyx 1 mm. long, very shallowly 5-lobate, the lobes broad, acutish, obscurely puberulent; corolla creamy white, 2.5 mm. long, glabrous, the lobes erect, with incurved tips, nearly as long as the tube; stamens shorter than the corolla lobes.

Type in the U. S. National Herbarium, no. 1,153,107, collected in wet forest at La Hondura, Province of San José, Costa Rica, altitude about 1,400 meters, March 16, 1924, by Paul C. Standley (no. 37892). Nos. 36555 and 37955, from the same locality also belong here.

The species is named for Don Otón Jiménez, well known as one of the enthusiastic botanists of Costa Rica, to whom I am under obligations for many favors received during a recent visit to Costa Rica. It was in his company that the type of the species was collected, during a visit to the rich valley of La Hondura, on the Atlantic watershed.

Psychotria orosiana Standl., sp. nov.

Shrub 1–3 m. high, much branched, the branchlets glabrous; stipules 5 mm. long, early deciduous, bilobate, the lobes subulate, ferruginous-tomentose, the sheath glabrous; petioles 2 mm. long or less; leaf blades oblong-obovate or elliptic-obovate, 6–12.5 cm. long, 2–4.5 cm. wide, rather abruptly acute or acuminate, with an obtuse tip, gradually or abruptly long-attenuate

to the base, thin, glabrous, paler beneath; inflorescence terminal, cymose-paniculate, long-pedunculate, the panicles 4–6 cm. broad, broader than long, many-flowered, open, the branches slender, glabrous, divaricate or ascending at a wide angle, the bracts and bractlets minute; flowers sessile or short-pedicellate, in few-flowered cymes; calyx glabrous, 1 mm. long, shallowly 5-dentate; corolla greenish white, glabrous, 2.5–3 mm. long, the lobes nearly as long as the tube.

Type in the U. S. National Herbarium, no. 1,153,182, collected in moist forest near Orosi, Province of Cartago, Costa Rica, March 30, 1924, by Paul C. Standley (no. 39803). The following specimens also represent the species:

COSTA RICA: Orosi, *Standley* 39766, 39768, 39861. El Muñeco, south of Navarro, Province of Cartago, alt. about 1,400 m., *Standley* 33444.

P. orosiana is well marked by the practically sessile leaves.

Psychotria siggersiana Standl., sp. nov.

Shrub, 1–1.8 m. high, the stems simple, green, sparsely villous-hirsute or glabrous; stipules green, about 7 mm. long, rounded-deltoid, usually glabrous, finally decidous from the persistent sheath; petioles 4.5–9 cm. long, hirsute or glabrate; leaf blades elliptic or broadly elliptic, 20–34 cm. long, 11–17 cm. wide, abruptly short-acuminate at apex or rounded and short-acuminate, acute at base, somewhat fleshy, copiously villous-hirsute on both surfaces, or sometimes glabrate above; inflorescence axillary, cymose-paniculate, many-flowered, ample, sometimes 20 cm. broad, the peduncles shorter than the cymes, the branches villous-hirsute; flowers sessile, usually densely clustered, the bracts large, green, villous, equaling or exceeding the flowers; calyx about 1 mm. long, sparsely short-villous, the 5 lobes deltoid-ovate; corolla 3 mm. long, greenish white, salverform, sparsely short-villous outside; fruit subglobose, red, 5–6 mm. long, the 2 stones deeply concave on the inner face.

Type in the U. S. National Herbarium, no. 1,153,058, collected in wet forest near Guápiles, Province of Limén, Costa Rica, altitude about 500 meters, March 12–13, 1924, by Paul C. Standley (no. 37210). The following additional specimens belong to this species:

COSTA RICA: Guápiles, *Standley* 37268, 37047, 37228, 37206. Tuis, alt. 650 m., *Tonduz*, 11465.

Related to *P. tonduzii*, but differing conspicuously in the broad leaves with copious pubescence. The species is named for Mr. Paul V. Siggers, of the United Fruit Company, who accompanied me upon two very agreeable and profitable collecting excursions to the Atlantic lowlands of Costa Rica.

BOTANY.—*New tropical American species of Urticaceae.*[1] ELLSWORTH P. KILLIP, U. S. National Museum.

Most of the species described in this paper belong to the genus *Pilea* and are based upon material collected in Central America. One new species of this genus from Mexico and five from South America, as well as a new *Pouzolzia*, are included. A species is transferred from the genus *Urera* to *Pilea*.

[1] Published by permission of the Secretary of the Smithsonian Institution.

Pilea tridentata Killip, sp. nov.

Herbaceous or suffrutescent, 40 cm. high or more, glabrous throughout; stem woody below, sulcate; stipules deltoid-ovate, 0.5 mm. long, deciduous; leaves penninerved with 6 to 8 pairs of nerves, opposite, those of a node conspicuously unequal and dissimilar, the larger 5 to 8 mm. long, 3 to 4 mm. wide, oblong-spatulate, 3-toothed at apex (or the upper occasionally subentire), tapering to a petiole 1 to 1.5 mm. long, the margins subequal at base or the upper side shorter, the smaller leaves 3 to 4 mm. long, 2.5 mm. wide, ovate or suborbicular, obtuse, sessile, very unequal at the base, the upper side oblique, the lower side auriculate, the upper surface densely, the under surface sparsely, covered with narrowly fusiform cystoliths 0.4 to 0.5 mm. long; plants dioecious (or monoecious?); staminate cymes 2 or 3-flowered, the peduncles 2.5 mm. long, the pedicels 2 mm. long, the perianth globose, 1.5 mm. in diameter, the tips of the segments acute, 0.3 mm. long; pistillate cymes 3 or 4-flowered, the peduncles 1.5 mm. long; pistillate flowers whitish, the middle perianth-segment elongate, 1.2 mm. long, the lateral segments 0.5 mm. long; achenes narrowly oblong, 1 mm. long, 0.6 mm. broad, minutely roughened.

Type in the U. S. National Herbarium, no. 799,669, collected near Cobán, Department of Alta Verapaz, Guatemala, 1,500 meters altitude, November 7, 1907, by H. von Türckheim (no. 2011). Türckheim's 2491, from the same locality, is also this species.

Pilea tridentata resembles *P. pleuroneura* Donn. Smith, but in that species the leaves are crenate to the middle, symmetrical at base, and proportionately longer.

Pilea argentea Killip, sp. nov.

Plant herbaceous, glabrous throughout; stems erect, up to 35 cm. (or more?) high, apparently simple, succullent, copiously covered with linear cystoliths; stipules broadly triangular-ovate, barely 1 mm. long, soon deciduous; leaves of a pair nearly equal, oblong, up to 8 cm. long, 3 cm. wide, acuminate, obtusish at apex, subauriculate and often slightly oblique at base, 3-nerved at base (nerves pale beneath, the lateral nerves reaching apex of blade), penniveined along midnerve, sessile or on petioles not more than 1 cm. long, entire at margin, thick, the upper surface green, densely covered with linear, straight or curved, and punctiform cystoliths, the under surface silvery-lustrous, with very numerous elongate-linear cystoliths on the nerves and conspicuous, elevated, curved or straight, linear cystoliths elsewhere; plants apparently diocecious; staminate inflorescence in axillary panicles near end of plant, the flowers in subglobose culsters, the perianth about 0.5 mm. long, pale proximally; pistillate inflorescence in panicles at nodes of upper half of plant, the flowers in few-flowered, subcontiguous, small clusters, the segments orbicular, subequal; achenes suborbicular, about 0.5 mm. long, narrowly winged, the surface minutely rugulose.

Type in the herbarium of the New York Botanical Garden, collected in forest east of Neiva, Department of Huila, in the Cordillera Oriental, Colombia, altitude 1500 to 2000 meters, August 1–8, 1917, by H. H. Rusby and F. W. Pennell (no. 654). Another specimen (no. 938), collected at the same time, also belongs to this species.

Pilea argentea is related to *P. parietaria* (L.) Blume, a common species of

tropical America. It may be distinguished by the sessile, or nearly sessile, proportionately longer and thicker leaves, and its coarser, more erect habit.

Pilea chiriquina Killip, sp. nov.

Plant glabrous, suffrutescent; stem decumbent and rooting at the base, the branches erect, 35 to 60 cm. high, simple; stipules deciduous; leaves triplinerved to apex, crenate-serrate, dark green above, paler beneath, the cystoliths of upper surface fusiform, minute; leaves of a pair very unequal, the larger oblong-lanceolate or oblanceolate, 4 to 5 cm. long, 1.2 to 2 cm. wide, caudate-acuminate, subauriculate at base (lower margin slightly the longer), the petioles 3 to 4 mm. long, the smaller leaves ovate or ovate-lanceolate, 1 to 1.3 cm. long, 0.6 to 0.9 cm. wide, acute, unequally subcordate at base (lower margin conspicuously auriculate), the petioles 1 mm. long; flowers dioecious, the pistillate borne in a compact flat-topped cyme 9 to 11 mm. wide, the peduncle 8 to 10 mm. long, the divisions of the perianth subequal, 0.9 mm. long, 0.5 mm. wide, bearing linear cystoliths on the outer surface.

Type in the U. S. National Herbarium, no. 1,010,199, collected in the humid forest along the upper Caldera River, 8 miles north of El Boquete, Province of Chiriquí, Panama, altitude 1,650 meters, February 11, 1918, by E. P. Killip (no. 3546). Duplicates of this collection are in the herbarium of the Rochester Academy of Science.

This species is to be distinguished from *Pilea pansamalana* by the longer peduncles of the pistillate cymes, by the subauriculate, rather than cuneate, leaf-bases, and by its erect, not prostrate, habit.

Pilea caudata Killip, sp. nov.

Glabrous throughout; stem erect, simple; stipules deltoid, 0.7 mm. long, 0.8 mm. broad, acute; leaves thickish, triplinerved, dark green above, paler beneath, bearing densely on the upper surface, sparsely on the lower surface, punctiform cystoliths, reticulate-veined (nerves and veins impressed and conspicuous beneath), cuneate at base, those of a pair conspicuously unequal, the larger lanceolate-elliptic, 10 to 15 cm. long, 2 to 4 cm. wide, caudate-acuminate (tips 2 to 3 cm. long), denticulate in upper third, the teeth 3 to 6 on a side, acute, 0.4 mm. long or less, the petiole 1 to 2.5 cm. long, channeled, swollen at base, the smaller leaves narrowly lanceolate-elliptic, 3 cm. long, 0.8 to 1 cm. wide, acuminate, entire or obsoletely serrulate at apex, the petiole 2 to 3 mm. long; plants dioecious?; staminate flowers in dense, axillary glomerules, globose, 0.8 mm. in diameter, the tips of the segments 0.2 mm. long, acute; pistillate flowers not seen.

Type in the U. S. National Herbarium, no. 408318, collected at Secoyocté, near Finca Sepacuité, Department of Alta Verapaz, Guatemala, April 14, 1902, by O. F. Cook and R. F. Griggs (no. 609).

The texture and venation of the foliage of this species resemble that of *Pilea riparia*. There is, however, a much greater diversity between the size of the leaves of a pair than in the case of *P. riparia*. The larger leaves, moreover, are much longer, the tips are shallowly, but distinctly and sharply denticulate, the margin is not thickened, and the cystoliths are uniformly punctiform, not linear at the margin of the blade.

Pilea donnell-smithiana Killip, sp. nov.

Erect herb, about 1 meter high, glabrous throughout; stem simple or few-branched near base; stipules minute, triangular-ovate, soon deciduous; leaves ovate or elliptic, crenate-serrate nearly to base, 3-nerved or subtriplinerved (nerves extending to apex of blade), rounded or subauriculate and often oblique at base, faintly marked with minute linear cystoliths on both surfaces; leaves of a pair conspicuously unequal, the larger 10 to 20 cm. long, 4 to 7 cm. wide, caudate-acuminate, their petioles 1.5 to 2.5 cm. long, the smaller leaves 1.5 to 3 cm. long, 1 to 1.5 cm. wide, acute, their petioles 0.3 to 0.5 cm. long; plants dioecious; staminate flowers borne in subglobose, distinct, usually pedicellate clusters in small axillary panicles 2 cm. long, or less, the perianth segments 0.5 mm. long; pistillate flowers in few-flowered subglobose heads about 5 mm. wide, the peduncles 2 to 3 mm. long, the middle perianth segment twice as long as the outer segments; achenes ovate, about 2.5 mm. long (large for the genus), strongly flattened.

Type in the U. S. National Herbarium, no. 799,603 (staminate); collected at La Palma, Province of San José, Costa Rica, altitude 1460 meters, September, 1898, by A. Tonduz (Herb. Inst. Phys.-geogr. Costa Rica no. 12,655; J. D. Smith no. 7467, distributed as *P. costaricensis*).

Additional specimens examined:

COSTA RICA: La Hondura, Prov. San José, 1300–1700 meters, *Standley* 37841, 37866, 37899.

PANAMA: Along Holcomb's trail, about 10 miles above El Boquete, Prov. Chiriquí, 1700 meters, *Killip* 3562.

The description of the pistillate inflorescence is based on Standley's 37841 (U. S. N. H. 1,229,574).

From *P. costaricensis* Donn. Sm. this new species differs in both staminate and pistillate inflorescence. The staminate flowers are in small distinct glomerules on short few-branched panicles; in *P. costaricensis* they are in very dense, nearly sessile cymes. The pistillate flowers are in globose, few-flowered heads, while in *P. costaricensis* they are in compact sessile cymes. The achenes of *P. donnell-smithiana* are fully twice as large as those of its near relative.

It is quite fitting that a species of Pilea should be named for Captain J. Donnell Smith, who has been one of the few botanists in the last half-century to study critically this interesting genus.

Pilea cornmanae Killip, sp. nov.

Plant herbaceous, erect, up to 60 cm. high, the stem branched, yellowish green, densely covered with elevated linear cystoliths, glabrous; stipules orbicular or ovate, about 5 mm. long, 3.5 mm. wide, obtuse, persistent, yellowish green, mottled with red; leaves coarsely serrate (teeth 2 to 3 mm. long, acutish), dark green above, paler beneath, densely covered on both surfaces with linear cystoliths about 0.3 mm. long, occasionally sparsely strigillose above with hyaline hairs; leaves of a pair unequal, the larger ovate-lanceolate, 3 to 7 cm. long, 1.5 to 3 cm. wide, acuminate at apex, acute and often oblique at base, the petioles 1.5 to 4 cm. long, the smaller leaves broadly ovate or suborbicular, 1.5 to 2 cm. long, 1 to 1.5 cm. wide, acute at apex, subrotund and oblique at base, the petioles 4 to 5 mm. long; plants monoecious (or dioecious), the heads unisexual, the staminate and pistillate often borne at the same axil; staminate flowers densely clustered in globose heads 9 to 10 mm. in diameter, the peduncle slender, 1 to 3 cm. long, the perianth globose

or subturbinate, 2 mm. wide, the segments long-caudate, erect; pistillate flowers loosely clustered in subglobose heads 4 to 5 mm. in diameter, borne on peduncles 5 to 7 mm. long, the perianth segments subequal; achenes broadly ovate or suborbiclar, about 1 mm. long.

Type in the U. S. National Herbarium, no. 1,010,196, collected in dense forest along the Río Caldera, 7 miles north of El Boquete, Province of Chiriquí, Panama, altitude 1650 meters, February 11, 1918, by Mrs. L. R. Cornman (Killip 3543). A duplicate of the type is in the herbarium of the Rochester Academy of Science.

Additional specimens examined:

COSTA RICA: La Hondura, Prov. San José, 1300–1700 meters, *Standley* 37779, 37822.

Because of the striking differences in size and shape of the leaves at each node this species should probably be placed in Weddell's section *Heterophyllae*. In general appearance, in the texture of the stem and leaves, in the large persistent stipules, the cystoliths, and the peculiar hyaline hairs on the upper leaf-surfaces, it bears a close resemblance to *P. auriculata* Liebm., a species of the group *Pubescentes Longipedunculatae*. The elongate segments of the staminate flowers and the differently shaped leaves clearly distinguish it, however, from P. *auriculata*.

Pilea rusbyi (Britton) Killip, comb. nov.

Urera rusbyi Britton, Bull. Torrey Club **28**: 310. 1901.

Pistillate plants of this specis in the Buchtien Herbarium, recently acquired by the National Museum, show that it is of the genus *Pilea* rather than *Urera*, and is related to P. *anomala* Wedd. Because of this additional material it seems advisable to amplify the earlier description.

Stipules ovate, 1 to 2 mm. long, acutish, connate; petioles up to 6 cm. long; leaves abruptly acuminate (acumen up to 1.5 cm. long), often oblique at base, 3-nerved or subtriplinerved, above bearing numerous punctiform cystoliths, nearly destitute of cystoliths beneath but punctate with inconspicuous dark dots; plants dioceious; panicles much branched, wide-spreading, 20 (or up to 30?) cm. wide; staminate inflorescences on peduncles up to 4 cm. long, the perianth segments ovate, 1 mm. long; pistillate inflorescences on peduncles up to 7 cm. long, longer than the accompanying petiole, the perianth segments ovate, subequal or the lateral two-thirds as long as the middle segment, the achenes ovoid, 1 mm. long, 0.8 mm. wide, acutish.

Specimens examined:

BOLIVIA: Yungas, 1800 meters, *Rusby* 1774 (type collection of *Urera rusbyi*). Polo-Polo, near Coroico, North Yungas, 1100 meters, *Buchtien* 3754.

Pilea pauciserrata Killip, sp. nov.

Low slender herb, glabrous throughout, the stem repent, at length erect, about 15 cm. high, the internodes 1 to 2 cm. long; stipules ovate, 1 mm. long, obtuse; leaves narrowly obovate, 6 to 20 mm. long, 2 to 5 mm. wide, cuneate-attenuate at base, coarsely serrate above middle (serrations 4 or 5 on each side, triangular, up to 1.5 mm. long, acute, the upper margin of each serration at right-angles to the midrib), entire at base, penninerved (lateral nerves faint), membranous, sessile or short-petioled (petioles up to 2 mm. long), bearing on upper surface, especially near margin, a few small, fusiform or

linear cystoliths, on under surface a few punctiform cystoliths; plants apparently dioecious; staminate flowers 3 mm. wide, purple above, pale at base, sessile or short-pediceled in few-flowered pedunculate heads (peduncles slender, up to 1.5 cm. long), the perianth segments ovate, 1 mm. long, mucronulate, bearing linear cystoliths on outside.

Type in the U. S. National Herbarium, no. 1,157,852, collected at Unduavi, North Yungas, Bolivia, altitude 3400 meters, November, 1910, by O. Buchtien (no. 2811).

This species resembles *P. serrulata* (Sw.) Wedd., having leaves of much the same shape. The leaves, however, are of thinner texture, and the cystolithic marking is wholly dissimilar. *Pilea pauciserrata* is distinctly herbaceous, having none of the shrubby habit of its relative.

Pilea gracilipes Killip, sp. nov.

Plant herbaceous, glabrous throughout; stem repent, the branches erect, simple, 8 to 40 cm. high; stipules minute, triangular-ovate, 1.2 to 1.5 mm. long, acute, deciduous; leaves ovate-lanceolate or elliptic, 1 to 8 cm. long, 0.7 to 2.5 cm. wide, acuminate at apex, rounded or acutish at base, crenate-serrate (teeth 0.5 to 1 mm. long, acute or often mucronulate), 3-nerved (nerves extending to upper third of blade), bright green above, paler and often glaucescent beneath, both surfaces bearing numerous faint linear cystoliths; leaves of a pair similar in form, subequal in size or one three-quarters as long as the other, the petioles of the smaller up to 1.5 cm. long, those of the larger about twice as long; plants monoecious or dioecious, the staminate and pistillate inflorescences often arising at the same axil; staminate flowers in globose, 12 to 20-flowered heads 5 to 7 mm. wide, the peduncles filiform, 2 to 3.5 cm. long, the perianth marked with linear cystoliths on outside, proximally yellowish, distally dark green, the tips of the segments barely 0.1 mm.; pistillate flowers in loose glomerules in interrupted spikes or racemose-paniculate, the peduncles filiform or linear, 2 to 5 cm. long, usually much exceeding the petioles, the middle perianth segment linear-oblong, 1 mm. long, cucullate, the lateral segments suborbicular, about half as long; achenes lance-ovate, 1 to 1.2 mm. long, 1 mm. wide at base, acute.

Type in the U. S. National Herbarium, no. 675,809, collected on a wet brushy slope in the humid forest at Los Siguas Camp, southern slope of Cerro de Horqueta, Province of Chiriquí, Panama, altitude about 1700 meters, March, 1911, by. W. R. Maxon (no. 5426).

The following specimens, all from Costa Rica, at altitudes varying from 1200 to 2500 meters, have also been examined:

Copey, *Tonduz* 11805, 11925. Tablazo, *Tonduz* 7927; Coliblanco, *Maxon* 309. Santa Clara de Cartago, *Maxon* 8165. La Estrella, Prov. Cartago, *Cooper* 384 (J. D. Smith 5950); *Standley* 39091, 39099, 39234, 39454. Cerro de la Carpintera, Prov. Cartago, *Standley* 34493, 35524. Volcán de Poás, *Standley* 34630; *Tonduz* 10790. Las Nubes, Prov. San José, *Standley* 38540, 38541, 38824.

Much of the material here cited was distributed as *P. auriculata*, a species with smaller rotund-rhombic leaves, pilose with hyaline hairs on the upper surface, large persistent stipules, and the middle segments of the pistillate flowers auriculate. *Pilea gracilipes* more closely resembles *P. dauciodora*, but is at once distinguished by larger, differently-shaped leaves and long-peduncled staminate flower-clusters.

Pilea angustifolia Killip, sp. nov.

Plant herbaceous, glabrous throughout; stem repent at base, erect, simple or few-branched, 40 to 45 cm. high; stipules deltoid, 0.8 mm. long, 1 mm. wide at base, deciduous; petioles enlarged at base, those of a pair usually unequal, the longer 4 to 8 mm. long, the shorter 2 to 3.5 mm. long; leaves narrowly lanceolate, 4 to 10 cm. long, 0.8 to 1.2 cm. wide, those of a pair similar in shape, subequal in size (or the one four-fifths as long as the other), triplinerved (nerves extending to upper third of blade), acuminate at apex, narrowed to a cuneate or subcordate base, serrulate (teeth acute, often mucronulate, 1 mm. long), bearing on both surfaces minute linear cystoliths; plants monoecious; staminate flowers densely congested in globose heads 0.6 to 1 cm. in diameter, the peduncles very slender, filiform, 3 to 4 cm. long, the pedicels 1 to 1.5 mm. long, the perianth segments with tips about 0.4 mm. long; pistillate flowers congested in sessile cymes 3 to 5 mm. wide, the middle segment linear-oblong, about 1 mm. long, the lateral segments ovate, 0.6 mm. long; achenes oblong, 1 mm. long, 0.7 mm. wide, acute, smooth.

Type in the U. S. National Herbarium, no. 473,969, collected at Juan Viñas, Reventazón Valley, Costa Rica, altitude 1000 meters, April 21, 1903, by O. F. Cook and C. B. Doyle (no. 181).

Related to *P. gracilipes*, this differs in its proportionately narrower leaves, nearly sessile pistillate cymes, and in the unusually slender peduncles of the large staminate heads.

Pilea chiapensis Killip, sp. nov.

Plant herbaceous, 30 cm. high or more, glabrous throughout; stem angulate, grooved, slightly flexuous; stipules early deciduous; leaves of a pair very unequal and dissimilar, the larger oblanceolate or oblong (upper often falcate), 7 to 11 cm. long, 2 to 2.5 cm. wide, acuminate at apex (acumen up to 2 cm. long), narrowed to an oblique base, remotely and irregularly serrulate along upper fourth of margin (teeth obtuse or acutish, somewhat callous-thickened), otherwise entire, 3-nerved to apex of blade, the petioles 3 to 5 mm. long, the smaller leaves ovate or ovate-lanceolate, 1.5 to 2.5 cm. long, 0.5 to 1 cm. wide, acuminate or acute at apex, narrowed at base, subsessile or with petioles up to 2 mm. long, 3-nerved to apex, entire or crenate-serrulate at apex, both kinds of leaves faintly marked with linear and punctiform cystoliths on upper surface, destitute of cystoliths but sparsely black-punctate on under surface; staminate cymes subsessile, densely flowered, 5 to 7 mm. wide, the flowers on pedicels about 2.5 mm. long, the perianth globose, 2 mm. wide, the teeth minute, barely 0.3 mm. long; pistillate cymes subsessile, 5 mm. long, the flowers sessile, the segments unequal; achenes ovate, 0.5 mm. long.

Type in the herbarium of the Academy of Natural Sciences, Philadelphia, collected at the junction of the Teapa and Amatán rivers, Chiapas, Mexico, August 6, 1890, by J. N. Rivirosa (no. 938).

This species should be placed among the *Heterophyllae*, though it is quite unlike any other species of that section. In many respects it resembles *P. mexicana* Liebm., of the section *Glabratae Brevipedunculatae*, which, however, has the opposite leaves equal.

Pilea pallida Killip, sp. nov.

Plant herbaceous, glabrous throughout, or the nerves of the very young leaves pubescent beneath; stem simple, 30 cm. high or more, stipules lanceolate, 3 mm. long 1.5 mm. wide, acute, densely striate without with linear

cystoliths; petioles 1.5 to 4 cm. long, those of a pair subequal; petioles 1.5 to 4 cm. long, those of a pair subequal; leaves elliptic-lanceolate, 8 to 12 cm. long, 3 to 4 cm. wide, caudate-acuminate (tip up to 2.5 cm. long), tapering to petiole, triplinerved to base of tip, serrate (teeth obtuse, often mucronulate, 2 to 4 mm. long), densely covered with fusiform and punctiform cystoliths, dark green above, paler, slightly silvery-lustrous beneath; plants dioecious; staminate flowers densely congested in sessile subglobose clusters 3 to 4 mm. wide, the perianth globose, the tips about 0.4 mm. long; pistillate flowers in much-branched flat-topped cymes, shorter than the petioles, about 1 cm. long, 1.2 cm. wide, in the axils of the upper leaves, the branches of the inflorescence and the outside of the perianth densely striate with linear cys toliths, the middle perianth segment linear, 1 mm. long, obtuse, the lateral segments ovate, 0.2 mm. long, acute, scariose at margin; achenes ovoid, about 1 mm. long, 0.8 mm. wide, acute, wing-margined, the surface black, minutely papillose.

Type in the U. S. National Herbarium, no. 1,036,444, collected at Sibubi Falls, Sixaola Valley, Panama, June 6, 1918, by W. W. and H. E. Rowlee (no. 376).

This species differs from *P. quichensis* Donn. Sm., to which it is clearly allied, in the shape, texture, and particularly the cystolithic marking of the leaves.

Pilea lippioides Killip, sp. nov.

Plant apparently suffrutescent, glabrous throughout, the stem branched, marked, especially at nodes, with linear cystoliths; stipules oblong-lanceolate, about 3 mm. long, 1.5 mm. wide, bearing elongate linear cystoliths, at length deciduous; petioles filiform, those of a pair unequal, the longer 3 to 4 cm. long, the opposite ones about half as long; leaves of a node essentially equal and similar, ovate-elliptic, 2.5 to 6 cm. long, 1.5 to 2.5 cm. wide, acuminate at apex (acumen about 5 mm. long), tapering to petiole, coarsely crenate-serrate to base (teeth subimbricate, mucronulate), penninerved (lateral nerves 5 to 10 pairs, impressed above, conspicuous beneath), the upper surface dark green, with numerous short linear cystoliths, the under surface paler, with linear cystoliths on the nerves and veins and punctiform cystoliths elsewhere; plants monoecious; staminate flowers in dense globose heads borne singly in the axils of the upper leaves on slender peduncles 1.5 to 2 cm. long, each head subtended by an involucre of 8 bracts, the outer 4 orbicular, about 6 mm. in diameter, the inner 4 oblong, about 4 mm. long, 2.5 mm. wide, obtuse, the bracts of the same texture and cystolithic marking as the stipules, and completely enveloping the flowers before anthesis; staminate perianth segments 3, about 4 mm. long, with long linear tips; pistillate flowers in small, loosely about 10-flowered, subglobose clusters on peduncles 5 to 10 mm. long in the axils of the lower leaves, the perianth segments subequal, triangular-ovate, about 0.8 mm. long; achenes suborbicular, 1.5 mm. long, smooth.

Type in the U. S. National Herbarium, no. 531,410, collected near Río Flautas, Río Paez Valley, Tierra Adentro, Department of Huila, in the Cordillera Central, Colombia, altitude 2900 meters, January 26, 1906, by H. Pittier (no. 1216).

This species is remarkable for the conspicuous involucre subtending the staminate inflorescence, the general aspect of the plant strongly suggesting *Lippia*. In Weddell's monograph[1] of the family it would come nearest *P.*

[1] DC. Prodr. 16[1]: 144.

serratifolia Wedd., an Ecuadorean species which, from description, also has a conspicuous involucre but which differs in venation and cystolithic marking of the leaves as well as in the shape of the staminate perianth segments.

Pilea buchtienii Killip, sp. nov.

Succulent herb, glabrous throughout, the stem at first repent, at length erect, about 20 cm. high, the internodes 3 to 6 cm. long; stipules ovate, 2 mm. long, 1 mm. wide, obtuse, soon deciduous; petioles 1 to 3.5 cm. long, succulent, canaliculate; leaves of a node similar and equal, ovate-elliptic, 8 to 12 cm. long, 3 to 6 cm. wide, acuminate (acumen up to 1.2 cm. long), tapering at base to petiole, doubly crenate-serrate, entire at base, triplinerved (nerves reaching to base of acumen), reticulate-veined (nerves and veins conspicuous beneath), the upper surface dark green, bearing numerous punctiform and minute fusiform cystoliths, the under surface destitute of cystoliths, punctate with numerous spots scattered among the nerves; plants monoecious, the staminate and pistillate flowers borne in glomerules in separate few-branched panicles often at the same node, the peduncles of both inflorescences subequal, 1.5 to 2 cm. long, slightly shorter than the subtending petioles; perianth segments of staminate flowers oblong, 1.2 mm. long, obtuse, the stamens slightly longer; perianth segments of pistillate flowers ovate, subequal, about 0.5 mm. long, concave; achenes conical, 1.5 mm. long, 1 mm. wide, strongly flattened.

Type in the U. S. National Herbarium, no. 1,156,963, collected at Antahuacana, Espíritu Santo, Bolivia, altitude 750 meters, June, 1909, by O. Buchtien (no. 4526).

Nearest to *P. marginata* Wedd., this species is distinguished by its proportionately wider leaves with margins doubly serrate, and by the longer peduncles. *Pilea marginata* is said to be dioecious, while *P. buchtienii* is monoecious. The foliage of this species, especially in the venation, resembles that of *P. rusbyi* (Britton) Killip, though the leaves are thicker and the margin differently cut. *Pilea rusbyi*, however, is dioecious, and the peduncles are much longer.

Pilea hitchcockii Killip, sp. nov.

Plant herbaceous, the stem repent below, erect, about 30 cm. high, slender, ferruginous-strigillose, especially above, leafy near summit, the internodes at middle and toward base 3 to 4 cm. long; stipules ovate-orbicular, 4 to 7 mm. long, 2 to 4 mm. wide, rounded or subtruncate at apex, sparingly pubescent, persistent; petioles up to 4 mm. long, strigillose; leaves of a node similar but often slightly unequal, narrowly elliptic, 2 to 7 cm. long, 0.7 to 1.5 cm. wide, acute or acuminate at apex, acute or rarely slightly rounded at base, coarsely crenate-serrate or often sinuate-serrate (teeth obtuse or acutish), pennineved (lateral nerves 12 to 15 on each side), the upper surface dark green and glabrous, faintly marked with numerous fine linear cystoliths, the under surface much paler, densely strigillose on the nerves, otherwise glabrous, densely covered with minute punctiform cystoliths; plants monoecious; the cymes unisexual (staminate at lower nodes, pistillate at upper), dichotomous, up to 4 cm. long (including peduncle), the flowers sessile or subsessile; perianth segments of staminate flowers ovate, 1.5 mm. long, acute, glabrous; perianth segments of pistillate flowers ovate-lanceolate, 0.5 mm. long, acute, the apex often reflexed; achenes broadly ovoid, 1 to 1.2 mm. long, the margin thickened.

Type in the U. S. National Herbarium, no. 1,195,706, collected on a tree stump, in the valley of the Pastaza River, between Baños and Cashurco, 8 hours east of Baños, Province of Tungurahua, Ecuador, altitude 1300 to 1800 meters, September 25, 1923, by A. S. Hitchcock (no. 21825).

This species, having much the general appearance of narrow-leaved forms of *Euphorbia heterophylla*, belongs with the long-peduncled, pubescent species of the section *Dentatae*, though it is apparently the only species with penninervation. The venation is similar to that of *P. fallax* Wedd., a species with dimorphic leaves, and of *P. abetiaefolia* Killip, a very distinct Colombian plant.

Pilea pittieri Killip, sp. nov.

Plant herbaceous, decumbent or erect, up to 40 cm. high, the stem simple or few-branched, glabrescent below, sparingly pubescent above, densely marked throughout with linear cystoliths; stipules linear-oblong, 5 to 6 mm. long, 2 mm. wide, obtuse, deciduous; petioles 1 to 5 cm. long, those of a pair subequal (or one 2 or 3 times as long as the other), pubescent; leaves ovate or ovate-lanceolate, 3 to 10 cm. long, 1.5 to 7 cm. wide (those of a pair similar but slightly unequal), long-acuminate at apex, rounded or subcordate at base, 3 (or occasionally 5)-nerved (inner lateral nerves three-fourths length of blade), reticulate, serrate or serrate-crenate nearly to base (teeth obtuse or acute, minutely undulate-crenulate), the upper surface dark green, glabrous, bearing numerous minute linear cystoliths, especially along the nerves, the under surface paler, densely pubescent on nerves and veins, punctate on veins, bearing less numerous similar cystoliths; plant monoecious (or occasionally dioecious?); staminate cymes solitary in the axils of the lower leaves or at the leafless nodes of the rooting portion of the stem, subsessile (or on peduncles up to 3 cm. long), pubescent, densely flowered, the perianth segments linear-spatulate, 2 to 3 mm. long, 1 to 1.2 mm. wide, striate on outside with linear cystoliths; pistillate spikes solitary in the axils of the upper leaves, 4 to 5 cm. long, the peduncles slender, glabrous, 2 to 4-forked, the flowers borne in subglobose clusters 3 to 4 mm. wide, the middle perianth segment 0.6 to 0.8 mm. long, cucullate, twice as long as the lateral segments; achenes ovate, 1 mm. long, 0.8 mm. wide, acute, flattened, unicostate at center of both faces.

Type in the U. S. National Herbarium, no. 1,080,422, collected along the Río de la Paz, Sarapiquí Valley, Costa Rica, altitude 1300 meters, May 5, 1901, by H. Pittier (Herb. Inst. Phys.-geogr. Costa Rica 14149).

Additional specimens examined:

Costa Rica: (Province Cartago) La Estrella, *Standley* 39204. Orosi, *Standley* 39730, 39760, 39811, 39862. El Muñeco, *Standley* 33943. (Province San José) La Hondura, *Standley* 37713.

This species is related to *P. acuminata* Liebm., a dioecious plant with staminate and pistillate inflorescences similar and having much larger leaves, and to *P. pubescens* Liebm., a species with androgynous spikes and with the upper leaf-surface strigillose.

Pilea standleyi Killip, sp. nov.

Plant herbaceous, the stem decumbent, at length ascending to about 20 cm., slender, pellucid, green, glabrescent below, pubescent above, particularly at the nodes; stipules suborbicular, 3 to 4 mm. long, rounded, persistent; petioles up to 3 cm. long, densely pubescent, those of a pair subequal; leaves

ovate-elliptic, 2 to 6 cm. long, 1.5 to 3 cm. wide, acute at apex, rounded or acutish at base, crenate-serrate nearly to base, 3-nerved (lateral nerves extending to upper fourth of blade), dark green and glabrous above, paler beneath, densely pubescent on the nerves, bearing numerous linear cystoliths on both surfaces; plants dioecious; peduncles of pistillate inflorescence up to 5 cm. long, in the axils of the upper leaves, slender, glabrous, dichotomous, the flowers borne in contiguous clusters at the ends of the branches, the perianth segments unequal, the middle segment about 0.7 mm. long, the lateral segments less than one-half as long, hyaline; achenes suborbicular, about 0.8 mm. long, narrowly wing-margined, not costate on faces.

Type in the U. S. National Herbarium, no. 1,229,585, collected at Las Nubes, Province of San José, Costa Rica, altitude 1500 to 1900 meters, March 20–22, 1924, by Paul C. Standley (no. 38697).

The principal character by which this species may be distinguished from *P. pittieri*, which it resembles in general appearance, is in the smaller, conspicuously wing-margined, ecostate achenes.

Pouzolzia phenacoides Killip, sp. nov.

Shrub 1 to 1.5 meters high, branched, the branches hirsutulous above, glabrate below; stipules lanceolate, 5 to 6 mm. long, acuminate, pilosulous on midrib without, soon deciduous; leaves ovate or ovate-lanceolate, 2 to 6 cm. long, 1.5 to 3 cm. wide, acuminate at apex, rounded at base, petiolate (petioles up to 1.5 cm. long, slender), coarsely dentate-serrate except in lower quarter, 3-nerved at base, sparsely strigillose above with stiff white hairs, appressed-pilosulous on the nerves beneath; plants monoecious, the flower clusters androgynous or unisexual; staminate flowers in small, axillary, 1 to 5-flowered clusters, short-pedicellate, the perianth 4-lobed, about 1.5 cm. long, pubescent without, slightly exceeded by the stamens; pistillate flowers 1 to 5, short-pedicellate or subsessile, at base of staminate inflorescence, the perianth tubular, about 2 mm. long, contracted at the short 4-toothed beak, about 12-nerved, finely puberulous without; achenes ovoid, about 2 mm. long, dark brown, shining.

Type in the U. S. National Herbarium, no. 1,229,521, collected on the Cerro de Piedra Blanca, above Escasú, Province of San José, Costa Rica, January 31, 1924, by P. C. Standley (no. 32484). Standley's 34679, from La Ventolera, on the southern slope of the Volcán de Poás, altitude 1700 meters, is also this species.

This is apparently the only known American species of *Pouzolzia* with toothed leaves. The flowers are much like those of *P. occidentalis* (Liebm.) Wedd. The general appearance of the specimens suggests *Phenax hirtus* or *Phenax mexicanus*.

PROCEEDINGS OF THE ACADEMY AND AFFLILATED SOCIETIES

WASHINGTON ACADEMY OF SCIENCES

183D MEETING

The 183d meeting of the ACADEMY was held on January 7, 1924, in the assembly hall of the Cosmos Club. The program consisted of brief addresses constituting a *Symposium on Coal*

Program: GEORGE OTIS SMITH, Director of the U. S. Geological Survey, *Coal a national issue.* Under a classification that is more practical than scientific there are two kinds of coal—the coal that we think we can not get along without, and the coal that we actually can not get along without; coal for which there are substitutes, and coal for which there are no substitutes; the luxury fuel, anthracite used by the favored few, and the industrial fuel, bituminous coal, used by the whole country. The one is used in the homes of a few States in the East; the other by the industries of the whole country.

We mine as much bituminous coal in a day as anthracite in a week. Of our anthracite resources one-fourth has been mined; of the bituminous resources less than one per cent. In recent years anthracite production has failed to keep pace with population; by way of contrast, bituminous consumption per capita has increased 10-fold in a lifetime.

Bituminous coal is indispensable because upon it rest transportation and industry. Interstate commerce is really what puts the "United" into United States, and American industry as we know it today surpasses that of other nations in the degree to which we strengthen the arm of labor with mechanical power. The 3¼ horsepower or more of prime mover capacity with which the average wage earner is equipped means an annual consumption of about 10 tons of coal per wage earner for power alone, and power generation is only one of the uses of coal in industry. The uninterrupted mining of coal and its transportation and country-wide distribution thus make up an indispensable service. Shut down the coal mines or stop the coal-laden trains and the whole country would soon be not only cold and dark but idle and hungry.

The uninterrupted operation of the coal mines is a vital function of working America, and it thus becomes a national issue whether or not the coal industry performs this service so as to meet the public need. Call it public utility or not, coal mining is absolutely essential to the general welfare.

As we discuss the function of the coal industry it is well to realize the magnitude of its daily task of supplying the country with fuel. An average day's output of the bituminous mines would load a coal train extending from Washington nearly to New Haven, and if to this is added the daily quota of anthracite cars the coal train would extend to Hartford.

Four years ago Mr. Tryon and myself described what is wrong with the coal industry by the phrase "Bad load factor." Unfortunately this load factor is not improved. The law of supply and demand, however, works out differently with anthracite and bituminous coal. The undercapacity of the anthracite mines to meet even the ordinary demand is in marked contrast with the overdevelopment of the bituminous mines whose capacity far exceeds even the peak demand. In the anthracite industry restraint of trade is unnecessary for the overdemand maintains price levels. In the bituminous industry no combination or monopoly of producers has ever controlled either supply or prices. The insistent demand from buyers is the immediate cause

of exorbitant prices whenever supply has been inadequate. Herein lies the opportunity of the consumer; to help stabilize production and prices by stabilizing demand.

Uncertainty and irregularity of demand are not the only causes of the present unfortunate condition of too many bituminous mines and too many miners. How wasteful is this overdevelopment may be seen in the fact that in 1921 the bituminous mines turned out a million less tons than in 1910 but employed 100,000 more men, a difference suggesting decreased skill on the part of the mine worker but in fact due wholly to the shorter working year—149 days in 1921 as against 217 days in 1910. This is not progress.

The outstanding cause of this wasteful use of labor and capital in the bituminous industry is the uncertainty of its labor supply. The present monopoly of mine labor has followed in the trail of the Union's necessary beneficial work in behalf of the mine worker. But that makes labor monopoly no less injurious to the general public, and its power needs to be curbed. The open threat of tying up the commerce of a nation and shutting down its industries is the menace of economic chaos and therefore the real issue today concerns the sovereignty of the American people in their relation to those who own coal mines or work in coal mines: Is the people's right to an uninterrupted supply of coal the dominant right? On this issue, President Coolidge has declared that "the public interest is paramount" and that selfish failure in service is "such a betrayal of duty as warrants uncompromising action by the government." (*Author's Abstract.*)

DAVID L. WING, *Operator's costs and profits.* The practical use made of cost-of-production information is twofold: to assist the operator in the intelligent conduct of his own business, and to furnish information required of him by the Government. One of these requirements has been due to the administration of the Federal Income and Excess Profits tax; the other to the position of coal as a basic industry, which caused Government regulation of it during the war, and to almost continuous investigation from 1917 on, by Congressional committees, or governmental commisssions.

Costs of production vary greatly from mine to mine, and from field to field, due partly to physical conditions, and partly to different conditions of labor, transportation facilities, investment in labor-saving equipment, management, etc. The sales realizations also show wide variations, due to the use made of the coal, the competitive conditions in the markets, etc. In fixing prices for a field under price regulation, the average cost must not be taken as a basis, but instead, a cost at the 75 or 80 per cent level of production, in order not to curtail needed output. Large margins can be sometimes obtained by low-cost operators who show sales realizations below the field average. Margins must not be confused with net profits to the operator as they do not show return on actual investment. (*Author's Abstract.*)

H. FOSTER BAIN, Director of the Bureau of Mines, *Hazards of the coal industry.* Among the costs of coal society must pay is the cost of death and disability among miners. Health hazards in mining are not greater than in industry in general; accident hazards apparently are greater than most but not all American undustries. While the loss of life per ton of coal is less than abroad, it is much larger in number per 1000 workers. This follows from the fact that we work thicker beds, which more frequently lie at low angles, but we use more machinery and have speeded up production. Part of the difference is in the national spirit, whichis venturesome; our own men are disposed to take chances. It will be impossible to prevent accidents entirely, but it is not impossible to decrease their number and to limit their extent in a

mine. This is especially true of the great mine disasters. It should also be possible to decrease the number of haulage accidents. For these things scientific and technical research is still necessary. To decrease accidents due to fall of rock, which account for half the deaths in mines, campaigns of education such as the Bureau of Mines first-aid instruction are needed in order to inculcate safety thinking by the men. (*Author's abstract.*)

F. G. TRYON, of the U. S. Geological Survey, *Overdevelopment of the bituminous coal industry and its cure.* In the bituminous coal industry there are 150,000 miners and 200,000,000 tons of mine capacity in excess of what can be continuously employed. This results in a higher cost of production, which must ultmately be borne by the public, and in unrest among the miners through intermittent employment. It makes difficult the negotiation of wage agreements, and is thus an indirect cause of strikes. Plans to reduce overdevelopment include requiring a license to open a new mine, distributing railroad coal cars to well-established mines in preference to speculative enterprises, permitting combination and coöperative marketing among producers, and encouraging voluntary agreements between operators and employers to provide for unempoyment insurance or for methods of wage payment that will encourage steady operation. All of these plans are shown to be open to serious objection, either on legal or economic grounds. The attitude of the industry is shown to be opposed to regulation and in favor of a policy of *laissez-faire.* Unless sweeping changes in existing laws are to be made, the means of reducing overdevelopment are confined to overcoming the specific causes that have promoted it, that is, by gradual improvement of the technique and business methods employed by the industry. (*Author's abstract.*)

CHARLES P. NEILL, Manager of the Bureau of Information of the Southeastern Railways, *Wages in the coal industry.*

O. P. HOOD, *Consumers' Economics.* Cheap fuel has always been available in America, leading to unthrifty habits in its use. Each new price level has made old equpment less satisfactory, and we find ourselves with ample room for improvement in the use of fuels. A change in attitude of the public mind towards fuel resources is needed. Various economies are being practiced by the several groups of consumers, namely, the railroads, the large industries, public utilities, the small industrial plants, and the domestic consumer.

The application of our scientific knowledge to the problem of combustion and heat absorption in the various industries is our most effective means of meeting advancing prices. The domestic consumer in the east will ultimately have to learn to use bituminous coal, and when really careful attention is given less tons of coal will be used and satisfactory service obtained, but with no more effort and thought applied than with anthracite the results will always be unsatisfactory. (*Author's abstract.*)

184TH MEETING

The 184th meeting was held jointly with the Geological Society and the Philosophical Society in the Auditorium of the Interior Department building on January 23, 1925. T. A. JAGGAR, Director of the Hawaiian Volcanological Observatory, gave two addresses, one on the Tokyo Earthquakes, and the other on the Hawaiian Volcanoes. These were illustrated by lantern slides and motion pictures.

The Tokyo Earthquake.—The earthquake of September 1, 1923, which produced the fires that so nearly destroyed Tokyo and Yokohama, had its epicenter, according to the Tokyo seismographs, in the middle of the oceanic depression near Oshima Island between the Awa and the Izu peninsula. The seismic damage, however, was not at its maximum on Oshima Island. The

bottom of Dagami bay, north of Oshima, showed changes in depth in places as great as 200 fathoms. It looks as if block-faulting had occurred in the bottom of the bay in coincidence with the earthquake, the sea-bottom north of Oshima showing a drop of 900 feet. The greatest seismic disturbance was at Manazuru Point on the west side of Sagami Bay, at Kamakura and Yokosuka on the Sagami Peninsula, and at Tateyama on the Awa Peninsula. These places all suffered intense vertical shaking that resembled an up and down pounding from below; at Manazuru this sort of violent disturbance was repeated again and again during twenty-four hours. The shore-line was lifted from 1 to 9 feet.

The dying away of the earthquake effects from Sagami Bay outwards is rapid towards the west in the hard volcanic rocks of the Izu region, and more gradual towards the northeast in the soft river deposits of Tokyo Bay. The center of actual damage was about the town of Yokosuka. The intensity of the damage is remarkably dependent upon topography and on quality of construction. In some of the districts where the damage was most severe, honestly built and new houses stood up in extraordinary fashion. One of the interesting features of the great earthquake is the apparently freakish way in which the damage was distributed. Some small villages were almost totally thrown down in the midst of a region where large neighboring farm houses stand almost uninjured. In Tokyo itself some persons in the suburbs thought that the earthquake was less severe than the one they had experienced in April, 1922. Beyond Tokyo to the north and east the damage by earthquake dies away very rapidly within two or three miles, and there is very little damage on the east side of the Awa Peninsula. According to reports from the mining districts on the west side of the Izu Peninsula, there were miners working underground who did not know that an earthquake had occurred, and they perceived no falling of materials in the stopes or other signs of disturbance. Indeed, in some places in the Suzenji district it is said that the shock was not perceived even on the surface. In Tokyo the earthquake effects were much stronger on the bottom land and near the canals than they were in the hilly suburbs, and in general the earthquake in Tokyo, could it have been measured on the solid rock, would not rank as a shock of very great magnitude. The damage due to the earthquake was enormously greater in Yokohama.

Fires were set in all the larger towns; this includes Ogowara, Kamakura, Yokosuka, Hojo on the Awa Peninsula, Yokohama, and Tokyo. Most of Yokohama was burned and about half of Tokyo. There was a high wind on the day of the earthquake and about 120 fires are reported to have started in Tokyo alone. The earthquake occurred at 11.59 a.m. when fires were burning in all kitchens; the wooden houses and the crowded blocks with traffic congestion and much gasoline make it impossible to deal with fire when many fires are started at the same time during a high wind. The situation was complicated by a change of wind from southwest at noon to northerly in the evening. This increased the loss of life, for crowds which had taken refuge to windward of the first fires found themselves to leeward, and in attempting to shift positions became surrounded and hopelessly lost. The loss of life will never be known accurately, as the official statistics involve large numbers of missing in addition to the known dead. The loss of life certainly greatly exceeded that of the Messina earthquake, 140,000, and may have been as large as 400,000.

In the last 21 years there have been eleven great disasters; the earthquakes of San Francisco, Valparaiso, Kingston, Messina, Cartago, Guatemala, Avezanno, and Tokyo, and the volcanic eruptions of the Caribee Islands,

Vesuvius, and Sakurajima. Lives lost have averaged 30,000 per year and property destroyed $500,000,000 a year. The increase of intense catastrophes is due to concentration of population, not to increase in the frequency or the violence of earthquakes or eruptions. Any of the cities of our Pacific coast may be visited by another severe earthquake, and the regions farther east can not be considered exempt, as is shown by the New Madrid earthquake of 1812, the Charleston earthquake, and the earthquakes of 1755 and 1797 in the northeastern part of the country. The 1755 earthquake at Boston was as strong as the Charleston shocks of 1886. Precaution should be taken accordingly, particularly with regard to the danger from the fires that an earthquake usually starts. The presence of automobiles in the streets and large stores of gasoline complicate the problem.

The Hawaiian Volcanoes.—The speaker gave a brief account of the geography of Hawaii with especial reference to its volcanoes, and described the work of the Volcano Observatory on Kilauea. Inside the crater the topography is continually changing. When there is a rapid rise of lava, it is generally around the edge of the crater as if the central part had been plugged by solidifying lava.

The hourly observation of the liquid lava discloses fluctuation of level, presumably tidal in character. There is a twice-a-day fluctuation of from two to seven feet, a daily variation of from three to five feet, and a monthly shifting of the times of maximum and minimum level. The hard lava floor of the crater also showed a daily change in level of about one foot, but the times of maximum and minimum were nearly opposite to those for the liquid lava. Seasonal tilting of the ground was also found of from 15 to 25 seconds of arc. Seasons in which this tilting was cumulative away from the center of island and of least monthly range showed the least volcanic activity, and a decline of the lava column both at Kilauea and Mauna Loa. It is indicated that the observations, particularly those of the tidal movements, will ultimately be of value in predicting future eruptions. (*Abstracts revised by author*)

185TH MEETING

The 185th meeting was held jointly with the Geological Society in the assembly hall of the Cosmos Club on February 21, 1924.

Program: ALFRED C. LANE, Professor of Geology at Tufts College, *The age of the earth and the oceans.*

There are at least 41 different ways of figuring age. The first group of methods includes those depending upon the loss of heat of the sun or the earth; they have ceased to have any great value at present owing to the uncertainty of the effect of radioactivity and the uncertainty as to the temperature from which to start. The second group may be called the ledger, or debit and credit, methods. The methods of this group depend upon the process of deposition and erosion which is always going on; the stuff that is eroded has to be deposited somewhere. Therefore there must be a balance, like a trial balance on books of account. Thus the lime is concentrated in limestones and the sodium goes into the salt of the ocean. The ocean, like any other great salt lake, has slowly acquired its sodium, while the water and the chloride have been furnished by volcanoes; the ocean is probably growing in volume. A separate account may be opened for each element and all these methods should check, as they do fairly well, indicating an age for the earth not far from 100,000,000 years.

On the other hand, the methods of attaining the age of the earth by atomic disintegration give much longer times. The atoms of uranium explode like kernels of corn, and if we knew the rate at which the corn popper was popping

and the amounts of popcorn made, we could know how long the machine had been running. Also in both cases helium gas is given off and finally in the process of popping, the atoms fly off with such velocity as to produce a halo of disturbance in the surrounding mica or other material. All these methods have been used to estimate the age of the earth, and ages as great as one thousand million to 15 hundred million years have been obtained.

The question arises at once how this discrepancy can be reconciled. Attention was called to the work of the Committee on Atomic Disintegration of the National Research Council as calculated to throw light on this important subject, as well as to have practical value in separating the granites of different ages. This would have at times commercial value. Perhaps the reason for the difference in ages is that in the ledger type of determinations we have to figure from the present rate at which processes take place. It is more likely that atoms have been exploding at the same rate indefinitely than that the forces in action upon the earth's crust have always been uniform, and it seems more probable that events are happening more rapidly nowadays than they have on the average in the past, so that the rate of accumulation of sodium and its transportation by the streams now is a good deal faster than it has averaged in times past.

WALTER D. LAMBERT, *Recording Secretary.*

THE PHILOSOPHICAL SOCIETY

918TH MEETING

The 918th meeting was held in the auditorium of the Cosmos Club, April 4, 1925. The meeting was called to order by President FLEMING at 8:15 with 35 persons in attendance.

Program: F. WENNER and F. M. SOULE. *Measurement of cyclic changes in electrical resistance,* (presented by DR. WENNER). (Illustrated by lantern slides and by experimental demonstrations.)

Three methods were described for measuring a cyclic change in the resistance of a conductor when this change can be definitely synchronized with a test current. These differ from the methods generally used in which the resistance is measured first under one set of conditions, and then under another. They also differ from the one direct method seldom used, in that the test current is not passed through a galvanometer.

In one of these methods an inductor is placed in series with the galvanometer while the conductor under investigation is placed in parallel with this combination. In another, the potential drop in the conductor is balanced by the potential drop in an adjacent arm of a Wheatstone bridge. In the third, the potential drop in the conductor is balanced by the potentiometer arrangement. Each depends for its action upon the rectification produced when an alternating or varying current is passed through a conductor in phase with the cyclic change in its resistance.

They are extremely sensitive and extremely selective. The sensibility is such that under favorable conditions, it is possible to detect a cyclic change in resistance when the amplitude of this change is no larger than 10^{-8} times the resistance of the conductor in which it occurs. The selectivity is such that it is possible to detect such a cyclic change in resistance, of the same frequency as and in phase with the test current, when the amplitude of this change is no larger than 10^{-3} or even 10^{-6} times the amplitude of other changes which may be occurring at the same time. Further, they have the advantage that they do not require exceptionally high grade nor in general specially constructed apparatus for their application.

Attention was directed to a number of applications of these methods and the mathematical relations which apply for some of the simpler arrangements. The possible application to an investigation of telephone loud speakers and the acoustical qualities of auditoriums was illustrated by an experimental demonstration. (*Author's abstract.*)

Discussion. The paper was discussed by Messrs. WHITE, HAWKESWORTH, FOOTE, TUCKERMAN, ECKHARDT, and HUMPHREYS.

I. G. PRIEST.—*Gray skies and white snow.* (Illustrated.) At the Boston meeting of the Optical Society of America, October, 1924 (and previously), A. AMES, JR., showed that reproduction, in a plane picture, of the accurate projection of an "objective scene" may result in an effect on the beholder of the picture which is neither natural nor artistic. MR. AMES has shown that much more natural pictures, which recall the visual impression of the "objective scene" more pleasingly and vividly, may be made by introducing certain aberrations into the picture.

In the oral discussion of MR. AMES' paper, I pointed out that a similar, paradox may exist between the "values" (A. H. MUNSELL, "*Color Notation*," pp. 18 and 112) in a picture and the brightnesses in the "objective scene." My present purpose is to substantiate this proposition by specific experiments and quantitative photometric data.

The following major propositions have been demonstrated by experimental data.

(1) In viewing an actual scene of snow-covered ground and overcast sky, the sky may be perceived as "gray" and the show as "white" even though the brightness (technical photometric sense) of the snow be much less than the brightness of the sky.

(2) The observer may be conscious that the sky is "brighter" than the snow in spite of the fact that he perceives it as "gray," and the snow as "white."

(3) This paradox cannot be wholly explained by assuming that the snow appears white because of contrast with neighboring dark objects.

(4) If a virtual image of the "gray" sky be brought into juxtaposition with the direct view of the "white" snow, the snow may appear "gray" and the image of the sky "white."

(5) A vertical wall illuminated by half of the sky on an overcast day may be perceived as "white" and the sky immediately above and beyond it as "gray" even though the sky be much brighter than the wall.

(6) Photographs which render the relative brightness of sky and white objects approximately as they would be measured photometrically may produce quite unnatural effects.

(7) To give a faithful rendition of the visual effect, the artist may be obliged to make the relative brightness on his canvas in reverse order of the corresponding brightnesses in the "objective scene." (*Author's abstract.*)

Discussion. The paper was discussed by Messrs. WHITE, IVES, HAWKESWORTH, CRITTENDEN, ECKHARDT, L. H. ADAMS, GISH, FOOTE, and others.

919TH MEETING

The 919th meeting was held in the auditorium of the Cosmos Club, April 18, 1925. The meeting was called to order by President FLEMING at 8:14 with 32 persons in attendance.

Program: C. W. KANOLT. *Non-flammable liquids for low temperature thermostats.* (Illustrated by charts.) Thermostats are usually constructed with a stirred liquid bath. When the bath is to be used at a low temperature the liquid must be one that will not freeze at the temperature employed. Such

liquids as gasoline, petroleum ether and toluol have commonly been used for this purpose. Several very serious accidents have occurred in laboratories when such flammable liquids were used, especially when liquid air has been used for cooling. The evaporation of liquid air leaves liquid oxygen, and the condensation of a combustible vapor in liquid oxygen produces an explosive. A mild explosion or the collapse of a glass vacuum vessel may throw ignited liquid upon the operator.

The purpose of the investigation has been to find liquids which will not burn, which have very low freezing points and are otherwise suitable for thermostat liquids. The materials tried are chlorine and bromine derivatives of methane, ethane and ethylene and mixtures of these derivatives. By mixing the substances in suitable proportions it is usually possible to obtain mixtures having lower freezing points than any of the constituents.

The eutectic compositions and temperatures of mixtures of two, three and four components have been determined, and some mixtures of five components have been investigated. Viscosity and corrosiveness have also been studied. Suitable non-flammable liquids have been found that can be used at temperatures down to $-150°$C. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. STIMSON and L. H. ADAMS.

E. W. WOOLARD.—*A problem in mathematical expectation and its bearing on statistics.* In order that the mind may grasp the significance of a great mass of numerical data, and detect the laws and relations, if any, concealed in the figures, special methods of representation and of analytical treatment, known as statistical, are necessary. A common procedure is to replace the original body of data by a small set of summary coefficients which resume in themselves all the features of the complete data that are essential or relevant to the purposes in hand.

The statistical coefficients pertaining to a single variable that are now in common use, such as the mean, the standard deviation, etc., characterize only the features of the frequency distribution. In many cases, however, including those in which statistical methods and the theory of "errors" are applied to meteorological problems, the order in which the values of a time-variable occur is of importance, and it becomes desirable to have a coefficient characterizing the *order of succession* in a sequence of values:

Conclusions seriously in error may be drawn if statistical formulae are applied to problems in which the conditions in Nature do not conform to those under which the mathematical formulae are valid. For example, the mean temperatures of consecutive days are not independent of each other whereas the theorems of the theory of "errors" presuppose, in general, independent events. GOUTEREAU and MAILLET, in 1906, showed that in a sequence of numbers drawn at random from a Gaussian distribution, the ratio of the mean variability to the mean deviation is equal to $\sqrt{2}$, where by the mean variability is meant the mean of the absolute values of the differences between consecutive numbers in the sequence.

The author has obtained a formula, readily adapted to numerical computation, which gives the value of the mean variability in a random sequence drawn from a frequency distribution of any form. A comparison of this theoretical value with the actual value in any given series will give some indication of whether or not the series is a random one. The formula has been tested experimentally. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. LAMBERT and TUCKERMAN.

P. R. HEYL. *Some thoughts on the inertia of energy.* For the discovery of the

principle of the inertia of energy we are indebted to EINSTEIN, who pointed it out about 20 years ago. He first deduced it from the special theory of relativity, but shortly showed that it might be deduced as a corollary of Maxwell's electromagnetic theory without introducing relativity at all. Its pedigree therefore is unimpeachably classical, though the concept itself is subversive of much that was held to be firmly established in the nineteenth century.

There is a parallel to be drawn between the inertia of energy and the mechanical equivalent of heat. The latter principle asserts that there is a heat equivalent of energy, 4.2×10^7 ergs per calory. The former doctrine asserts similarly that there is a mass equivalent of energy, 9×10^{20} ergs per gram. The difference in dimensions between the erg and the gram is made up by the fact that the numerical coefficient (9×10^{20}) has the dimensions of the square of a velocity, being in fact the square of the velocity of light.

The great importance of the concept from a philosophical point of view is that it correlates the two fundamental concepts of matter and energy. It also shows potential energy in a new aspect, always accompanied by a slight change of mass, thus removing it from the "imponderables."

In his general theory of relativity EINSTEIN still further elaborates this doctrine of inertia of energy, showing that it leads to the concept that matter and energy are two different aspects, static and kinetic, of the same thing, space curvature. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. TUCKERMAN, HAWKESWORTH, and SILSBEE.

920TH MEETING

The 920th meeting was held in the auditorium of the Cosmos Club, May 2, 1925. The meeting was called to order by President FLEMING at 8:19 with 36 persons in attendance.

Program: F. NEUMANN. *Earthquakes of 1925—the problem of determining epicenters.* (Illustrated by lantern slides.) The Quebec earthquake was less severe than several others occurring in Asia during the early part of 1925. This is based particularly on the ground movements registered on the seismograph at the Honolulu Magnetic Observatory. In the United States the records of the Quebec earthquake of February 28th are complex, making a preliminary determination of the epicenter very uncertain.

The effect of earthquake waves on movable bodies in the disturbed region surrounding the epicenter is discussed with special reference to resonance effects and the doubtful possibility of using this kind of information to locate a definite epicenter.

The necessity of compiling more data concerning the variations in the velocity of earthquake waves is stressed. The Honolulu Observatory is cited as a station where particularly wide variations are in evidence.

The Alaskan earthquake of February 23d is cited as an instance in which five observatory records give fairly satisfactory results regarding both location and time of origin. It is shown how, at four observatories in the United States, several interpretations are possible in determining the epicenter of the Quebec earthquake. No positive statement is made concerning its exact location. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. LAPORTE and HECK.

H. D. HOLLER. *A method of studying electrode potentials and polarization.* (Illustrated by lantern slides.) A resistance-coupled electron-tube amplifier was used to operate an oscillograph for observing the variations in the potential of an electrode without requiring any current from the electrode under investigation. By making oscillographic photographs of the electrode potential at

the instant of interrupting a direct current through the cell under investigation it was possible to obtain a continuous record of the variation in potential. In cases where the polarization potential disappeared slowly enough, distinction could be made between the electromotive force of polarization and potential difference due to resistance.

By superposing alternating current upon the electrode under investigation and the half cell, it was possible with the use of the amplifier and a separately excited wattmeter, to measure the true alternating potential across the electrode. From this potential and the current passed through the cell, the boundary resistance at the electrode was computed from Ohm's law. This was permissible because the potential and current, as shown by the oscillograph were in phase, there being a negligibly small capacity effect.

The boundary resistance varied considerably with the different kinds of electrodes. The resistance of platinized platinum, lead, and carbon in sulphuric acid was small; that is, several hundredths of an ohm, while that of smooth platinum and copper in the same electrolyte was high, several tenths of an ohm. The resistance of copper in copper sulphate and zinc in zinc sulphate was also small. The results show that in some cases the electrode potential as measured by the potentiometer when current is flowing is more than 0.2 volt too high because of the potential due to resistance. In the cases where the boundary resistance was high, it decreased rapidly as the current was increased.

N. H. HECK. *The path of sound waves through sea water.* (Illustrated.) At first thought it might appear that a sound wave due to an explosion at a point off-shore would take a straight path to a hydrophone near the shore, and therefore the velocity would correspond to that of the warm surface layer. In tests made of the radio acoustic method of the Coast and Geodetic Survey in which the sound wave from an explosion of a small bomb 30 to 50 miles off-shore arrives at the hydrophone with sufficient energy to automatically send a return signal from a shore radio station to which the hydrophone is connected by cable, it was found that the velocity averaged considerably lower than for the surface layer. It was found further that it did not correspond to the straight line path, taking the earth's curvature into effect.

Tests made in July, 1924, off the coast of Oregon showed that the average temperature was 7°.1C. as the mean of surface to bottom temperature at several sections between the explosion and the shore. The corresponding velocity from Table 13, Special Publication No. 108, Coast and Geodetic Survey, is 1475 m. per second, while the measured velocity, based on visual determination of the position of the ship which fired the bomb, was 1473 m. per second. In October the corresponding figures were: average temperature 10°.9C., and the corresponding velocity 1491 m; measured velocity 1493 m.

It was shown by means of a diagram that a part of the wave front in the warm surface layers would travel faster and therefore the wave surface would be disturbed. Rays drawn from the bomb explosion normal to the successive wave fronts were bent downward; accordingly for successive wave fronts the energy per unit of area would be less near the surface than further down. For this reason, above a certain distance, perhaps ten miles, there would not be sufficient energy from the direct wave in the surface to send the signal. On the other hand, the waves which strike the bottom are reflected. For paths near the vertical the reflections are numerous and there is loss of energy at each reflection. It would appear then, that having given position of explosion and hydrophone, the required amount of energy reaches the hydrophone by a path which is based on maximum energy for a minimum number of reflections. As the depth of the water is small compared to distance, the actual path makes

very little difference in distance determination. The indication that the theory advanced is correct is upheld by the agreement of measured velocity with the theoretical velocity for the average temperature, top to bottom, and it is further upheld by the fact that in some cases a shoal projecting well above the bottom, but by no means up to the path of the straight line from bottom to hydrophone, actually blocks the transmission. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. VAUGHAN, W. M. DAVIS, STEPHENSON, MARMER, and HAWKESWORTH.

By request. PROF. W. M. DAVIS gave a formal talk on the *Significance of the discovery of a shoal in China sea*, and pointed out the bearing of this discovery on Darwin's theory of coral reefs.

921ST MEETING

The 921st meeting was held in the auditorium of the Cosmos Club, May 16, 1925. The meeting was called to order by President FLEMING at 8:16 with 34 persons in attendance.

Program: L. B. TUCKERMAN. *Full walled sturdy columns in theory and practice.* (Illustrated by lantern slides.) Sixty-nine large columns of H-shaped section, part rolled in one piece and part fabricated from plates and angles or from channels, were tested at the Bureau of Standards in coöperation with the American Bridge Company and the Bethlehem Steel Company.

The cross sectional areas were approximately 35 and 85 square inches, the lengths 12, 18 and 24 feet and the slenderness ratios ranged from 38 to 92. All were tested as "flat end" columns in the Bureau's ten million pounds vertical testing machine.

To determine the properties of the material over 1,000 tensile coupons were tested, so that the material in these columns was better known than in any other series of similar large columns.

The tests results indicate that the strength of solid walled sturdy column within this range of slenderness depends only slightly on the manner of construction, slenderness ratio or small accidental eccentricities, either of structure or of test conditions.

The column strengths, however, correlate closely with the weighted average yield point as determined by the coupon tests, indicating that the strength of the columns is determined largely by the properties of the material in the neighborhood of the yield point.

A study of various details of the behavior of the columns in the tests, especially the anomalous lateral deflections, and the "pick-up" of load, indicate that the behavior of these columns can be best explained by means of the "double modulus" theory, first proposed by CONSIDERE, developed and experimentally confirmed by KARMAN on small specimens, and later independently proposed by SOUTHWELL. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. L. J. BRIGGS, TRUSCOTT, and PAWLING.

I. C. GARDNER and F. A. CASE. *Photographing the interior of a rifle barrel.* (Presented by MR. GARDNER.) (Illustrated by lantern slides.) In order to study the progress of the erosion in the 0.30 caliber service rifle and machine gun barrels a camera has been designed for photographing the interior surface of the barrel. The apparatus consists essentially of a periscope of unit magnification of such dimensions as to permit entry into the bore of the rifle. A small electric lamp, also placed in the bore of the gun, provides the illumination. The periscope projects an image of a small portion of the bore on a strip of motion picture film. The barrel to be photographed is slowly drawn along the periscope

and simultaneously the film is moved at such a rate that there is no relative motion between image projected by periscope and film. If the barrel is drawn its entire length along the periscope, one obtains a picture on the film showing a strip of the interior of the bore of the same length as the barrel and including approximately one-sixth of the circumference. Six such pictures provide a photographic record of the entire surface of the bore upon which the detailed defects arising from the erosion can be clearly seen. The instrument is also excellently adapted for a visual examination of the interior of the barrel. A microscope magnifying 20 or 30 diameters can be used to view the image in the focal plane, and the different characteristics of the surface of the bore stand out clearly in good contrast. This camera is intended to be used to study the manner in which different steels resist erosion. (*Author's abstract.*)

Discussion. The paper was discussed by MESSRS. SILSBEE and HAWKESWORTH.

922ND MEETING

The 922nd meeting was a special meeting held jointly with the Washington Academy of Sciences and the Chemical Society of Washington, May 28, 1925, in the auditorium of the Cosmos Club. It was called to order at 8:30 by Dr. L. H. Adams, president of the Chemical Society of Washington, with 110 persons in attendance.

Program: HERBERT FREUNDLICH. *The state of aggregation and shape of colloidal particles.* (Illustrated by lantern slides.)

The X-ray has been applied to the study of colloids, and by its use we have been able to decide whether particles of a sol are crystalline or amorphous-solid (or liquid). It has been found that the particles of very many sols are crystalline, e.g. sols of gold, silver, Al_2O_3, and Fe_2O_3.

The state of aggregation of particles depends on the velocity of their formation; there is according to HABER a competition between a *grouping velocity* and an *orientation velocity*. If the period of precipitate formation is very short, definite orientation does not take place. The particles remain in a state of disorder and the precipitate is amorphous as in the case of quickly precipitated $Al(OH)_3$ and $Fe(OH)_3$. But where Al_2O_3 or Fe_2O_3 sols are formed by hydrolysis and if the tendency to crystallize is great enough, there is time for the process of orientation and consequently the particles crystallize.

The shape of amorphous particles is, as a rule, spherical.

Non-spherical particles, due to their shape, exhibit manifold phenomena. One of these is that of pronounced scintillation of light, i.e., not a continuous radiation of light as in the case of spherical particles. V_2O_5 is a characteristic sol of this type. The aged sol shows particles with a definite rod-like structure. The particles are crystalline. In old and non-coagulated concentrated sols the particles do not lie irregular, but in cloud-like swarms. Such groups are made up of very small colloidal particles. When such swarms are viewed through an ultra-microscope using a so-called azimuth diaphragm, long thread-like particles standing nearly parallel to one another may be seen. They possess weak scintillations caused by the Brownian movement. Old and concentrated Fe_2O_3 sols show this same phenomenon still more pronounced. But their particles are lamellar, and they settle down in regular spaced layers and an iridescent green color is reflected or refracted similar to the iridescent light of butterfly wings.

Double refraction is also a property of the sols composed of non-spherical particles. A simple explanation of this property is that these sols contain small crystals which are double refracting. Double refraction is only visible

in dilute sols, when the particles are arranged in a regular order. This may be achieved by different methods.

Theoretically, double refraction is correlated with dichroism. Many sols, especially the sols of V_2O_5 and Fe_2O_3, do not only exhibit a strong double refraction, if the particles are oriented in a regular way, but also show a pronounced dichroism. The change of double refraction and dichroism with the wave-length of light are correlated by a general rule.

The non-spherical shape of colloidal particles is not always caused by this crystalline state. In the case of latex, the sap of the india-rubber tree, for instance, the particles are pear shaped because a solid skin encloses an inner viscous fluid. This structure seems to be important for the stability of latex and there also seems to be a parallelism between their structure and the nerve of the india-rubber product. (*Author's abstract.*)

H. A. MARMER, *Recording Secretary.*